GLOBAL OPTIMIZATION DESIGN OF QMF FILTER BANKS *

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ABSTRACT

In this paper, we present a new global optimization method for designing QMF (quadrature mirror filters) filter banks. We formulate the design problem as a nonlinear constrained optimization problem, using the reconstruction error as the objective, and the stopband ripple, stopband energy, passband ripple, passband energy and transition bandwidth as constraints. This formulation allows us to search for solutions that improves with respect to the objective, and that performs better than or equal to the best existing designs with respect to the constrained measures. We present NOVEL, a global optimization method we have developed for solving nonlinear continuous constrained optimization problems, and apply it to find improved designs. We also show that relaxing the constraints on transition bandwidth and stopband energy will lead to significant improvements in the other performance measures.

1. INTRODUCTION

The design of digital filter banks is important as improvements can have significant impact in many fields. Among various filter banks, QMF FIR filter banks are an important class of filter banks that have been studied extensively.

In general, the design objectives of filter banks can be classified into two types, the first defining the overall performance of the filter bank, and the second defining the performance of each individual filter. Figure 1 summarizes the various design objectives used in this area for measuring the quality of a design. It is clear that the design problem is a multi-objective, continuous, nonlinear optimization problem, and a good design should have small distortions, small ripples, small attenuations, and short transition.

In this paper, we present a new global constrained optimization method for designing QMF filter banks. In a two-band QMF filter bank, the reconstructed signal is [4]:

$$\begin{aligned}\hat{X}(z) &= \frac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]\, X(z) \quad (1)\\ &+ \frac{1}{2}[H_0(-z)F_0(z) + H_1(-z)F_1(z)]\, X(-z).\end{aligned}$$

where $X(z)$ is the original signal, and $H_i(z)$ and $F_i(z)$ are, respectively, the response of the analysis and synthesis filters. To perfectly reconstruct the original signal based on $\hat{X}$, we have to eliminate aliasing, amplitude, and phase distortions. QMF FIR filter banks implement perfect reconstruction by setting $F_0(z) = H_1(-z)$, $F_1(z) = -H_0(-z)$

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Filter	Design Objectives
Overall Filter Bank	Minimize amplitude distortion (E_r)
	Minimize aliasing distortion
	Minimize phase distortion
Single Filter	Minimize stopband ripple (δ_s)
	Minimize passpand ripple (δ_p)
	Minimize transition bandwidth (T_t)
	Minimize stopband energy (E_s)
	Maximize passband flatness (E_p)

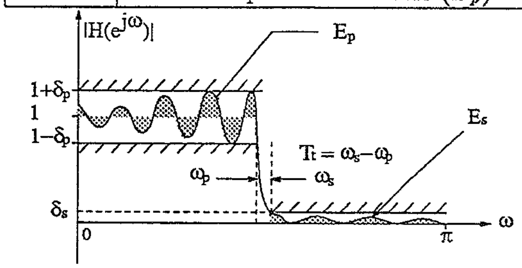

Figure 1. Possible design objectives of filter banks and an illustration of the design objectives of a single low-pass filter. ($[0, \omega_p]$ is the pass band; $[\omega_s, \pi]$, the stop band; $[\omega_p, \omega_s]$, the transition band.)

and $H_1(z) = H_0(-z)$, leading to one prototype filter $H_0(z)$ in the system. Let $\mathsf{h}(n)$ be the filter parameters. If $\mathsf{h}_0(n)$ is symmetric, then $\mathsf{h}_1(n) = (-1)^n \mathsf{h}_0(n)$ is antisymmetric, and the system has linear phase. This leads to zero aliasing and phase distortions in QMF filter banks, converting the design problem into an optimization problem that finds $\mathsf{h}_0(n)$ in order to

- minimize the amplitude distortion (reconstruction error) of the overall filter bank, and
- maximize the performance of the individual prototype filter $H_0(z)$.

2. NONLINEAR OPTIMIZATION OF QMF FILTER BANKS

The design of QMF filter banks can be formulated as a multi-objective unconstrained optimization problem or as a single-objective constrained optimization problem.

2.1. Multi-Objective Formulation

A possible multi-objective unconstrained formulation is to optimize the design with respect to a subset of the measures defined in Figure 1.

$$Min \quad E_r \text{ and } E_s \qquad (2)$$

where $E_r = \int_{\omega=0}^{\frac{\pi}{2}} (|H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega-\pi)})|^2 - 1)^2 \, d\omega$

and $E_s = \int_{\omega=\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega$

where ω_s is the stopband edge.[1]

Unfortunately, optimal solutions to the simplified optimization problem are not necessarily optimal solutions to the original problem. Oftentimes, performance in the objective measures not included in the formulation are compromised. For example, when E_r and E_s are the objectives to be minimized, the solution that gives the smallest E_r and E_s will probably have a large transition band.

In general, there is no method that can optimize simultaneously all the objectives in a general nonlinear multi-objective problem. One approach is to optimize a weighted sum of all the objectives and to solve the new problem using unconstrained nonlinear optimization methods [7, 3, 15, 1, 10]. For this approach to succeed, the weights on each objective must be chosen properly, and solving such a problem amounts to designs that trade one performance measure with respect to another.

Other approaches try to exploit specific features of the design problem. Examples include spectral factorization and heuristic methods [8, 16].

2.2. Single-Objective Constrained Formulation

In a single-objective constrained formulation, constraints are defined with respect to a reference design. The specific measures constrained may be application- and filter-dependent [15].

Constraint-based methods have been applied to design QMF filter banks in both the frequency [7, 1, 2, 8, 12, 14] and the time domains [9, 13]. In the frequency domain, the most often considered objectives are the reconstruction error, E_r, and the stopband ripple. As stopband ripples cannot be formulated in closed form, stopband attenuation is used instead (represented as E_s in (2)). In the time domain, Nayebi [9] gave a time-domain formulation with constraints in the frequency domain and designed filter banks using an iterative time-domain design algorithm.

In this paper, we formulate the design of a QMF filter bank in the most general form as a constrained nonlinear optimization problem, using the reconstruction error as the objective and other measures (stopband ripple, stopband energy, passband ripple, passband energy and transition bandwidth) as constraints:

$$\begin{aligned} &\mathit{Minimize}\ \ E_r \\ &\text{subject to}\ \ E_p \le \theta_{E_p} \qquad E_s \le \theta_{E_s} \\ &\qquad\qquad\quad \delta_p \le \theta_{\delta_p} \qquad\ \ \delta_s \le \theta_{\delta_s} \\ &\qquad\qquad\quad T_t \le \theta_{T_t} \end{aligned} \tag{3}$$

where θ_{E_p}, θ_{E_s}, θ_{δ_p}, θ_{δ_s} and θ_{T_t} are constraint values obtained in the best known design. The goal here is to find designs whose constrained performance measures are better than or equal to those of the reference design, and whose objective measure is better than that of the reference design. Since the objective and constraints are nonlinear, the problem is multi-modal with many local minima.

[1] Note that in QMF filter banks, E_r is non-zero. A multi-rate filter bank that enforces perfect reconstruction ($E_r = 0$) can be formulated as a constrained optimization problem with a goal of minimizing E_s [6, 5].

Finding global optimal solutions of nonlinear continuous constrained problems is one of the most challenging tasks in optimization. There are three global optimization approaches to solve (3).

Methods in the first approach use local search to determine local minima, and focus on bringing the search out of a local minimum once it gets there. The two classes of methods and their drawbacks are as follows.

- Deterministic methods, such as covering methods and generalized descent methods, do not work well when the search space is large.
- Probabilistic methods are weak in either their local or global search. For instance, gradient information is not used well in simulated annealing and evolutionary algorithms. In contrast, gradient descent algorithms with multistarts and random probing are weak in their global search strategy.

In the second approach, all constraints are absorbed into the objective function and weighted by penalty coefficients. These constant, selected ahead of time, are used to penalize the objective function when constraints are violated. The approach is not effective because it is usually hard to choose appropriate penalty values when constraints are violated.

In the third approach, the single objective constrained problem is solved by minimizing a Lagrangian function, which is the sum of the objective function and the constraints weighted by Lagrange multipliers. Although this is similar to a penalty-based method, it is more powerful because Lagrange multipliers are dynamically adjusted in order to push the search towards a feasible region.

In a Lagrangian formulation, a local minimum in a feasible region is called a *saddle point* at which the objective function is at a local minimum and the weighted sum of the constraints is at a local maximum. By using this property, saddle points can be found by *local search methods* that perform gradient descents in the original-variable space and gradient ascents in the Lagrange-variable space.

Since a saddle point is only a local minimum in a feasible region, *global search methods* are needed to bring the search out of a local minimum. Strategies like random restarts are not effective because the search space is too large to be covered. In the next subsection, we describe an effective global search strategy that relies on an external force to pull the search out of local minima.

2.3. NOVEL Global Optimization Method

NOVEL (*Nonlinear Optimization via External Lead*) is a global optimization method we have developed to solve nonlinear unconstrained optimization problems [11]. Starting from a Lagrangian formulation, our implementation of NOVEL has two stages.

In the *global search stage*, NOVEL looks for good starting points for the local-search stage. This is important because it first identifies good starting points before applying expensive local searches. This avoids repeatedly determining unpromising local minima as in multi-start algorithms and applying computationally expensive descent algorithms from random starting points. The result of this stage is a trajectory on the Lagrangian function space. The dynamics of the trajectory is controlled by two forces: local gradient to pull the trajectory towards a local minimum, and the force exerted by a gradient-independent trace function to pull the trajectory out of a local minimum. The latter is particularly important because it provides a continuous means of going from one local region to another, avoiding

Table 1. Performance of three QMF filter banks found by NOVEL as compared to that of Johnston's designs [7]. The objective to minimize is E_r, and others parameters are constrained with respect to Johnston's.

Performance	24D		32D		48D	
	NOVEL	Johnston's	NOVEL	Johnston's	NOVEL	Johnston's
E_r	3.660166e-05	4.861841e-05	4.965934e-06	5.941763e-06	5.623691e-07	5.936320e-07
E_s	2.564074e-04	2.564074e-04	3.860062e-05	3.860063e-05	6.892256e-07	6.892257e-07
E_p	3.348576e-06	4.358370e-06	3.563868e-07	5.829183e-07	2.679881e-08	3.544864e-08
δ_s	3.263258e-02	3.263258e-02	1.473594e-02	1.473594e-02	2.481871e-03	2.481871e-03
δ_p	4.124381e-03	4.124381e-03	1.727426e-03	1.834884e-03	4.808339e-04	4.808339e-04
$\Delta\omega$	0.539900	0.539900	0.503369	0.503369	0.472107	0.472107

problems in methods that determine new starting points heuristically and losing valuable local information found in a local search.

In the *local search stage*, NOVEL uses promising starting points identified in the global search stage and applies local searches to find saddle points in the Lagrangian function space. These local searches include gradient descents in the original-variable space and gradient ascents in the Lagrange-variable space. The designs found correspond to designs whose constraints are satisfied and whose objective is at a local minimum.

3. EXPERIMENTAL RESULTS

We have applied NOVEL to solve some QMF filter-bank design problems formulated by Johnston [7]. In our designs, we have used the reconstruction error as our objective to be minimized, and have constrained other performance measures with respect to those of Johnston's designs. Our goal is to find designs that are better than Johnston's results across all six performance measures.

To apply NOVEL, all the measures defined in Figure 1 and their gradients must either be in closed form or evaluated accurately by numerical methods. We have derived closed-form formulae for the reconstruction error, passband energy, stopband energy, and the corresponding gradients. The ripples, the transition band, and their gradients are found by numerical methods.

Table 1 shows the performance of 24D, 32D and 48D QMF filter banks designed by our method as compared to those of Johnston's. In all three cases, our designs have smaller reconstruction errors and passband energies, while all other measures are either better than or equal to Johnston's. Note that other design methods generally perform trade-offs, resulting in designs that are better in one or more measures but worse in others.

Figure 2 depicts our 24D QMF and Johnston's designs. It indicates that our design has smoother passband response and lower reconstruction error.

Note that Johnston used sampling in computing energy values whereas NOVEL used closed-form integration. Hence, designs found by Johnston are not necessarily at the local minima in a continuous formulation. To demonstrate this, we applied local search in a continuous formulation of the 24D design, starting from Johnston's design. We found a design with a reconstruction error of 3.83E-05, which is better than Johnston's result of 4.86E-05. By applying global search, NOVEL can further improve the design to result in a reconstruction error of 3.66E-05.

To summarize, performance improvements in NOVEL come from three sources. First, the closed-form formulation used in NOVEL is more accurate than the sampling

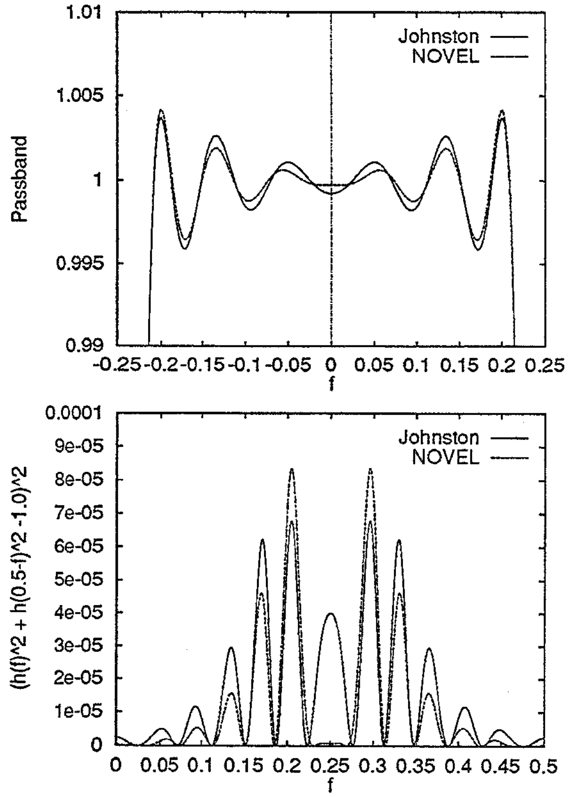

Figure 2. A comparison of our 24D QMF design and Johnston's design. The upper graph shows the passband frequency responses, and the lower graph shows the reconstruction errors.

method used in Johnston's approach. Local optima found by NOVEL are true local optima whereas Johnston's solution are local optima in a discrete approximation of the design problem. Second, NOVEL uses a constrained formulation which allows it to find designs that are guaranteed to be better than or equal to Johnston's design with respect to all performance measures. Third, NOVEL employs effective global optimization strategies that allows it to explore a large part of the search space without first committing to many expensive local searches.

By using our constrained formulation, we can further study trade-off in designing QMF filter banks in a controlled environment. Tightening the constraints in (3) will cause the reconstruction error to increase, whereas loosening them will lead to smaller reconstruction error.

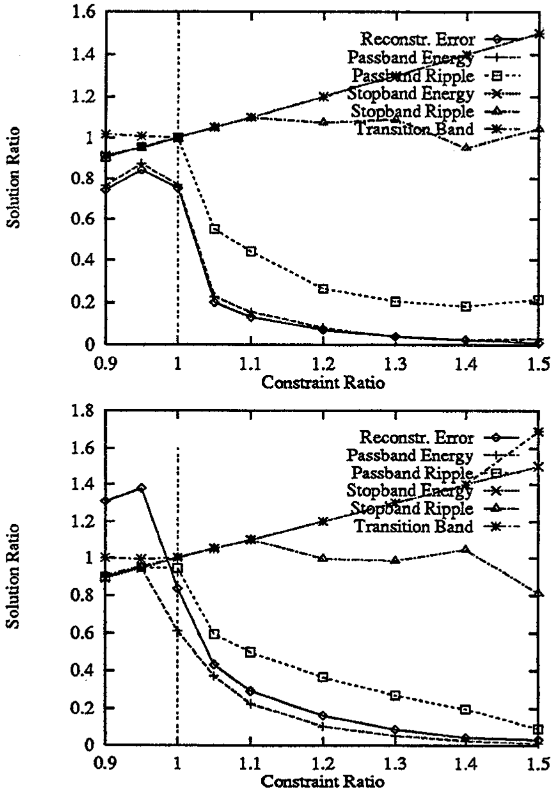

Figure 3. Experimental results in relaxing and in tightening the constraints with respect to Johnston's designs for 24D (upper) and 32D (lower) QMFs. The x-axis shows the ratio of the constraint in NOVEL with respect to Johnston's value. The y-axis shows the ratio of the measure found by NOVEL with respect to Johnston's.

Figure 3 demonstrates these trade-offs for 24D and 32D QMF filter banks. In our experiments, we have used Johnston's designs as our baselines. We use constraint ratio, R_c, to represent the tightening or the relaxation of all the constraints. For example, $R_c = 1.1$ means that *all* the constraints in NOVEL are set to 1.1 times of the measures in Johnston's design. The y-axis shows the solution ratio, which is the ratio between the measure found by NOVEL and that of Johnston's. Hence, the line $x = y$ in Figure 3 indicates that the resulting measure is exactly at the constraint value.

When constraints are loosened, the reconstruction error, passband energy, passband ripple and stopband ripple decrease significantly with respect to the relaxed constraints. These improvements are at the expense of the transition bandwidth and stopband energy, which increase according to the relaxed constraints.

When constraints are tightened, we have difficulty in satisfying the constraint on the transition bandwidth, and have found designs whose transition bandwidth is about the same as that when $R_c = 1$. This indicates that Johnston's designs are already very good, and that it may only be possible to have trade-offs between the transition bandwidth and the stopband energy.

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