Finding Good Starting Points For Solving Nonlinear Constrained Optimization Problems by Parallel Decomposition

Soomin Lee **and Benjamin Wah**
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Department of Electrical and Computer Engineering
and the Coordinated Science Laboratory

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

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Definition: Constrained Optimization Problems (COPs)

(P) : $\min_{\mathbf{z}} f(\mathbf{z})$ subject to $\mathbf{h}(\mathbf{z}) = \mathbf{0}$ and $\mathbf{g}(\mathbf{z}) < \mathbf{0}$,

where $\mathbf{z} = (\mathbf{x}, \mathbf{y})$: Variables, $\mathbf{x} \in \mathbb{R}^v$, $\mathbf{y} \in \mathbb{D}^v$
$f(\mathbf{z})$: objective function
$\mathbf{h}(\mathbf{z}) = (h_1(\mathbf{z}), \ldots, h_m(\mathbf{z}))^T$: Equality constraints
$\mathbf{g}(\mathbf{z}) = (g_1(\mathbf{z}), \ldots, g_r(\mathbf{z}))^T$: Inequality constraints

- NLP (Nonlinear Programming) and MINLP (Mixed Integer Nonlinear Programming)
- We focus on finding CLM (Constrained Local Minima).

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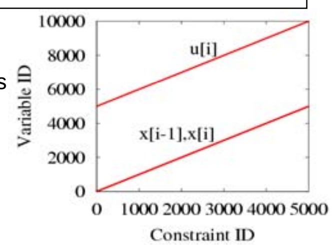
Motivation: Good Starting Points

- Ex: HAGER4 from CUTEr
  - An NLP with 10,000 variables and 5,000 constraints
  - A large problem that cannot be solved by an existing solver MINOS from the default starting point.

$$
\begin{array}{rl}
\min_{x,u} & \sum_{i=1}^{n}(C_1 z_{i-1} x_i^2 + C_2 z_{i-1} x_i (x_{i-1} - x_i) \\
 & + C_3 z_{i-1}(x_{i-1} - x_i)^2) + \sum_{i=1}^{n} C_4(u_i^2) \\
\text{subject to} & h_i(x,u) = 0 \\
\text{where} & \\
 & h_i(x,u) = (n-1)x_i - n x_{i-1} - \exp(t_i) u_i, \text{ for } 1 \le i \le 5000
\end{array}
$$

  - Very well structured
  - Constraints are all in the same format
    - Good constraint locality wrt variables



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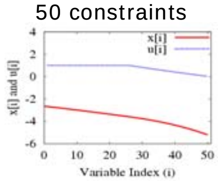
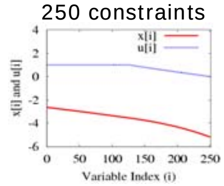
Motivation: Good Starting Points

- Relaxed problems of HAGER4 by constraint sampling

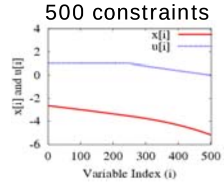
50 constraints



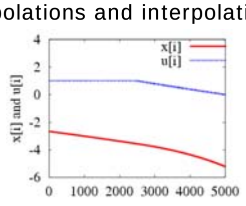
250 constraints



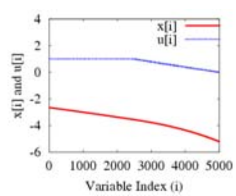
500 constraints



- Generated starting points (by extrapolations and interpolations)



- The final solution generated by other solver



Now, MINOS can solve the problem in 3.13 sec with this starting point.

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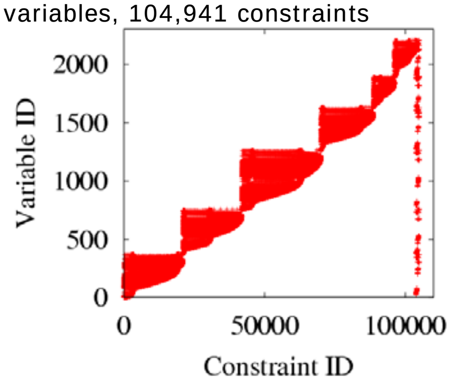
Constraint Localities in AI Problems

- Rovers-Propositional-P18 from IPC-5.
  - 2,207 variables, 104,941 constraints



- Constraint localities allow starting points to be generated

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Problem Statement

- Exploit the localities of constraints in constrained optimization problems
- In order to generate much simpler subproblems
- Whose solutions can be generalized to form good starting points to the original problems
- That allow the original problems to be efficiently solved

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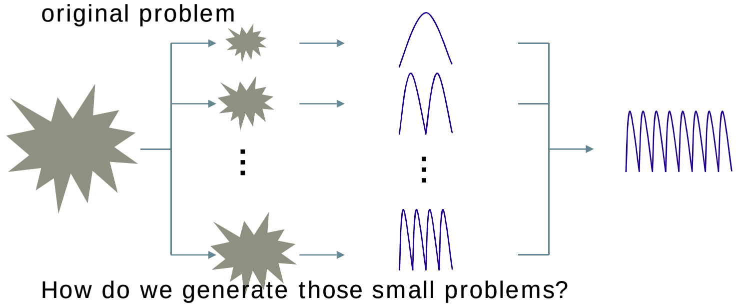
Approach: Generating Good Starting Points: Overview

1. Analyze the structure of the original problem
2. Generate small problems (relaxed/easier versions) that contain all the features of the original problem
3. Solve those small problems
4. Use the outputs to generalize to the starting point of the original problem

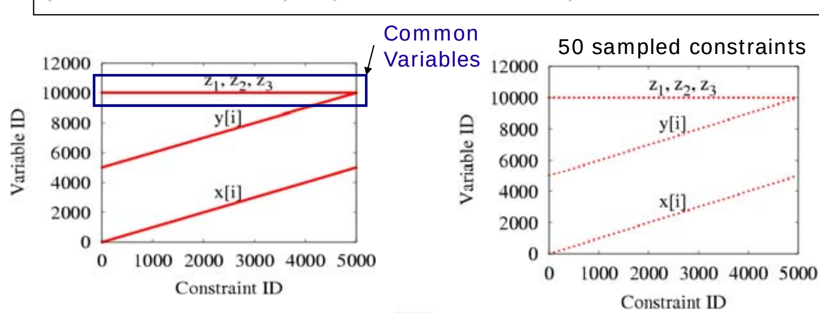


- How do we generate those small problems?
  - constraint relaxation

(A) With a Few Common Variables (1/2)

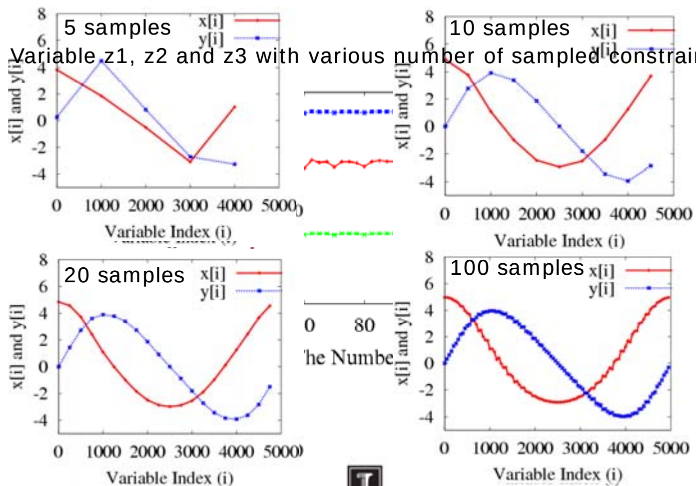
- Constraint sampling and interpolations
  - All the constraints share a few common variables
  - Example

$$\left((x[i]-z_1)^2+(y[i]-z_2)^2\right)^2-\left((x[i]-z_1)^2+(y[i]-z_2)^2\right)(1+z_3^2)^2=0,\ 1\le i\le 5000$$



(A) With a Few Common Variables (2/2)

Extrapolate variables x[i] and y[i] from subproblem with sampled constraints



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(B) With Coupled Constraints (1/3)

- Constraint sampling and extrapolation
  - Constraints are related to their neighborhood constraints
  - Example: $\boxed{(n-1)x_i - nx_{i-1} - \exp(t_i)u_i = 0, \text{ for } 1 \le i \le 5000}$

$$(n-1)x_1 - nx_0 - \exp(t_1)u_1 = 0$$
$$(n-1)x_2 - nx_1 - \exp(t_2)u_2 = 0$$
$$(n-1)x_3 - nx_2 - \exp(t_3)u_3 = 0$$
$$(n-1)x_4 - nx_3 - \exp(t_4)u_4 = 0$$
$$\cdots$$
$$(n-1)x_{5000} - nx_{4999} - \exp(t_{5000})u_{5000} = 0$$

  - Take a subset of contiguous constraints and extrapolate

$$(n-1)x_i - nx_{i-1} - \exp(t_i)u_i = 0, \text{ for } 1 \le i \le 500$$

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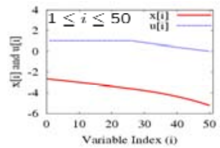
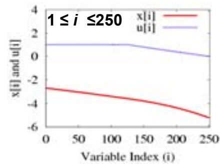
(B) With Coupled Constraints (2 / 3)

Example: $(n-1)x_i - nx_{i-1} - \exp(t_i)u_i = 0$, for $1 \leq i \leq 5000$

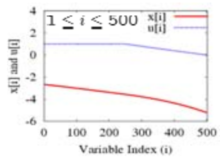
Simplified Problem #1



Simplified Problem #2



Simplified Problem #3



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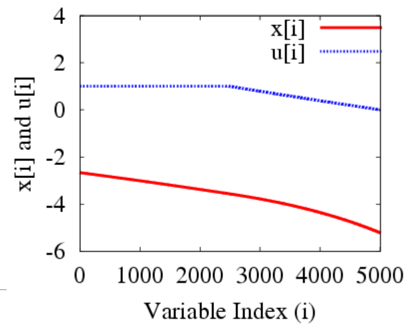

Generated Starting Point
$1 \leq i \leq 5000$

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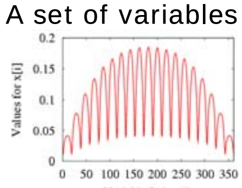
(B) With Coupled Constraints (3 / 3)

Example: A set of variables $x[1..N^2]$, $N = 59$

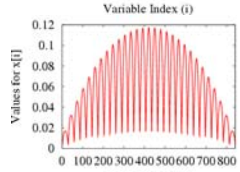
Simplified Problem #1

N = 19



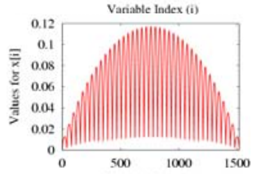
Simplified Problem #2

N = 29



Simplified Problem #3

N = 39



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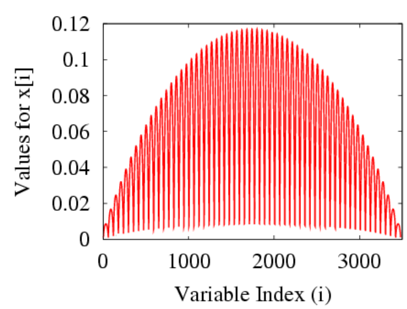

Generated Starting Point

N = 59

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(C) With Mixed Integer Constraints

- **Constraint relaxation for MINLPs**
  - Solve an MINLP as an NLP without the integrality of integer variables.
  - Apply (A) and (B) if MINLP is still too large to be handled by existing NLP solvers.

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Illustration on Performance of Approach

Name	Nc	Nv	Ns	Tech	Solver	Default		Proposed		
						Time	Sol	Sptime	Time	Sol
Ex4	3717 (3481)	7198	514	B	Conopt	f	f	5.61	27.26	0.08
					Ipopt	1.36	0.08	1.71	1.77	0.08
					Knitro	1.42	0.08	1.64	1.17	0.08
					Snopt	130.40	0.08	8.31	8.16	0.08
cont5_2_4	40200 (200)	40400	131	B	Conopt	152.80	0.07	3.02	55.36	0.07
					Ipopt	208.18	0.07	2.90	148.26	0.07
					Knitro	483.15	0.07	29.78	257.17	0.07
					Snopt	f	f	2.96	2180.97	0.07

For SNOPT, even a good starting point is not good enough.

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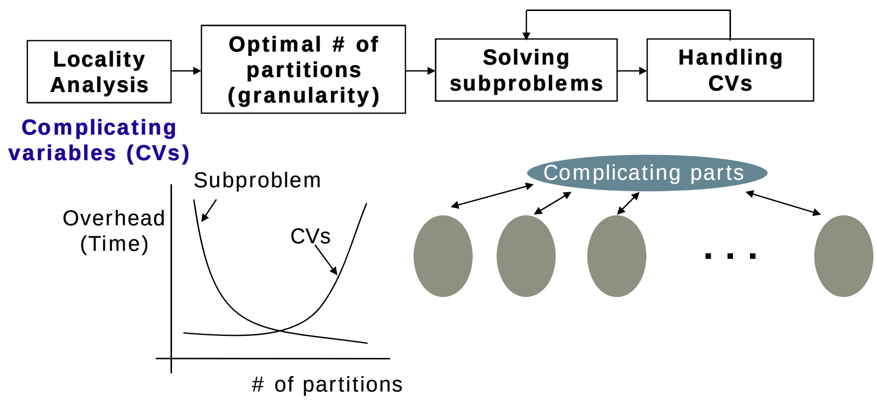
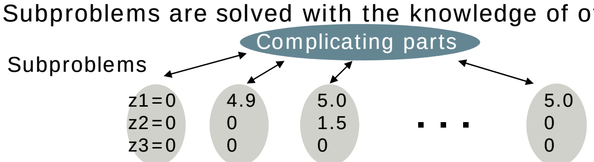
Handling Large-Scale COPs by Parallel Decomposition [CP2005, Wah/Chen]

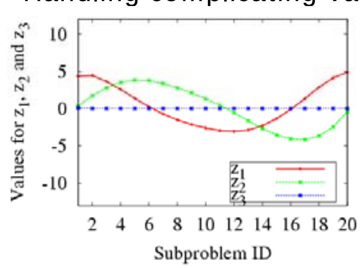


Again, Need Good Starting Points in Parallel Decomposition

- Fast convergence
  - Subproblems are solved with the knowledge of others.



- Handling complicating variables (CVs)



Need to enforce consistency of complicating variables in all subproblems.

Ex) Make them fixed throughout the solving process

Need good values for z1,z2 and z3

Illustration on Parallel Decomposition with Good Starting Points [CP2005, Wah/Chen]

Name	Nc	Nv	Ns	Tech
orthrgds	5000(5000)	10003	5,10,20	A
lukvli7	4(4)	50000	52,502	B

Name	Solver	Default w/o partitioning		Proposed w/o partitioning			Proposed w/ partitioning			
		Time	Sol	sp Time	Time	Sol	Np	Itr	Time	Sol
orthrgds	Conopt	439.41	5585.15	0.99	178.75	1523.90	25	9	131.50	2586.61
	Ipopt	3.15	1523.90	1.25	1.75	1523.90	2	1	2.96	1523.90
	Knitro	9.19	1776.23	1.50	1.75	1523.90	2	1	3.28	1523.90
	Snopt	f	f	1.12	f	f	20	1	18.58	1543.25
lukvli7	Conopt	35.71	62521.38	7.13	4.93	42643.98	5	1	12.48	-20731.69
	Ipopt	7.57	-18633.85	4.01	11.59	-18633.85	2	1	5.49	-20731.69
	Knitro	43.49	-18633.85	5.10	10.22	-18633.85	2	1	4.94	-20731.69
	Snopt	1966.72	-18633.85	139.74	1471.94	-18633.85	5	1	601.45	-20731.69

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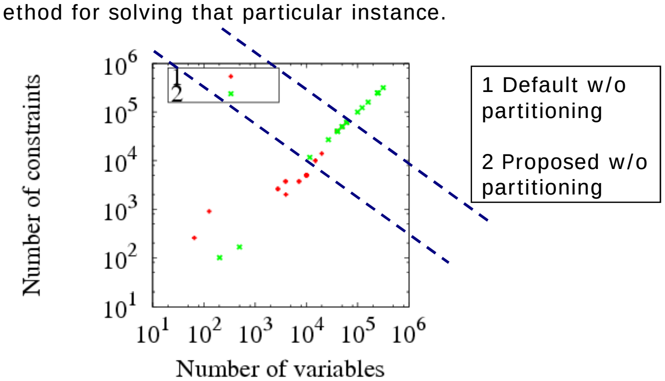
No Enumerations! - Problem Identification

- Each dot shows an instance.
- The best method for solving that particular instance.
- Ex) Knitro

Good starting points are important for large problems



- The same behavior applies to the other solvers as well.

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Conclusions/Future Work

- We were able to solve all the problems using one of the methods.
- We can determine what technique to use before actually solving a problem.

- Integrate the process of generating good starting points as a preprocessor to existing NLP and MINLP benchmarks.

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