The Evaluation of Partitioned Temporal Planning Problems in Discrete Space and its Application in ASPEN

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Abstract. In this paper, we study the partitioning and evaluation of discrete-space temporal planning problems and illustrate our techniques on ASPEN, an objective-based planner developed at the Jet Propulsion Laboratory for the automated planning and scheduling of complex spacecraft control operations. We formulate these planning problems as single- or multi-objective dynamic optimization problems and propose the necessary and sufficient extended saddle point condition (ESPC) that governs the correctness of locally optimal plans. We then decompose the ESPC into partitioned ESPC, one for each stage, and show that they are necessary and sufficient collectively. By utilizing these partitioned conditions, we present efficient search algorithms whose complexity, despite exponential, has a much smaller base as compared to that without using the conditions. Finally, we demonstrate the performance of our approach by integrating it in the ASPEN planner and show significant improvements in CPU time and solution quality on some spacecraft scheduling and planning benchmarks.

1 Introduction

Many planning and scheduling applications can be formulated as nonlinear constrained dynamic optimization problems with variables that evolve over time. In this paper we focus on single- and multi-objective temporal planning problems that can be formulated using a discrete planning horizon, discrete state vectors representing positive and negative facts, and constraints representing preconditions and effects of actions. We study the partitioning of these problems into subproblems related by global constraints, and develop methods for resolving the global constraints after the subproblems have been solved.

Consider ASPEN [4], an objective-based planning system for the automated planning and scheduling of complex spacecraft operations. It involves generating a sequence of parallel low-level spacecraft control commands from a set of high-level science and engineering goals [8]. Using a discrete time horizon and a discrete state space, an ASPEN model encodes spacecraft operability constraints, flight rules, spacecraft hardware models, science experiment goals, and operations procedures. It defines various types of schedule constraints that
may be in procedural form among or within the parallel activities to be scheduled. Such constraints include temporal, decomposition, resource, state-dependency, and goal constraints. In addition, the quality of a plan is defined in a preference score, which is a weighted sum of multiple preferences (that may also be procedural) to be optimized by the planner. Preferences can be related to the number of conflicts, the number of actions, the value of a resource state, and the value of an activity parameter.

Figure 1 illustrates a toy example planning problem from ASPEN. It involves scheduling four activities over a discrete horizon of 60 seconds in order to satisfy various constraints that relate the activities and power, resource, color, and state, and color changer, while trying to minimize the total power resource used. By indexing variables by time $t = 0, \ldots, 60$ Boolean variable $s_i(t)$ (resp. $a_i(t)$ and $c_i(t)$) that defines an activity (resp. starts and ends) at $t$. Let $c_i(t)$ be the color changer activity at $t$, which can be set to red, blue, green, or not active; $e_i(t)$ be the color state that can be set to red, blue, green; and $p_i(t)$ be the power supply at $t$.

The following illustrates a small portion of the constraints encoded, where $A \implies B$ can be translated into constraint $(1 - A) + B \geq 0$, and the objective is to minimize $\sum_{t=0}^{T} w(t)$:

- $a_i(t+1) \implies c_i(t) = 2$; // color state constraint for act.1
- $e_i(t) \neq c_i(t) \implies c_i(t-1) = c_i(t); // color state transition constraint
- $u_i(t) \leq p_i(t) \leq 25,$ $t = 0, \ldots, 60; // power resource capacity constraint
- $e_i(t+1) = 1 \implies a_i(t+1) \geq 0$ \iff $a_i(t) = 0$; // act.1 ends before start of act.3 by $[0,30].$ Using this formulation, the problem has a total of 198 constraints and one objective function.

Figure 2 illustrates another example that shows the 3,687 constraints of an initial (infeasible) plan generated by ASPEN [4] in solving CX1-PREF with 16 orbits.

Figure 2: The 3,687 constraints of an initial infeasible plan generated by ASPEN in solving CX1-PREF with 16 orbits.
new planners to solve each subproblem, pruning techniques in existing planners, such as constraint propagation and schemes for handling exceptions in execution, can be employed. Further, new planners developed in the future can be integrated easily in our approach.

Existing methods for solving discrete problems do not explore the partitioning approach because there is no effective method except expensive trial and error for resolving violated global constraints after the partitioned subproblems have been solved. For a similar reason, existing planners do not exploit partitioning in their search. These planners rely on global information and do not have mechanisms to combine the solutions of partitioned subproblems into global solutions.

Existing planners based on systematic searches explore the entire state space and are not amenable to partitioning because their locally feasible or optimal plans in a partitioned space may not satisfy all the global constraints. These include UCPOP [15], an early goal-directed planner based on the Partial Order Causal Link (POCL) technique; Graphplan [1] that searches a planning graph in order to minimize the length of a parallel plan; STAN [13], an efficient implementation of Graphplan; PropPLAN [5], a planner based on a naive breadth-first search of ordered binary decision diagrams; and System R [12], a systematic search method based on regression that solves one goal at a time.

Existing planners based on local searches employ heuristic guidance functions to search in discrete space. They do not work well on partitioned plans because their guidance heuristics are computed over the entire horizon in order to estimate the distance from a state to the goal state. Examples include HSP [2], a hill-climbing search using heuristic values obtained by solving a relaxed problem; FF [6], an enhanced hill-climbing search using heuristic values obtained by solving a relaxed Graphplan problem; AIAH [14], a hybrid planner on top of STAN and HSP; GRT [18] (and its extension to MO-GRT [19]), a two-phase planner that first estimates the distances between domain facts and goals, before searching by a simple best-first strategy; and ASPEN [4], a repair-based local-search method that can handle discrete temporal and metric constraints and that optimizes multiple objectives in a weighted sum.

Last, planners based on transformations convert a problem into a constrained optimization or satisfaction problem before solving it by existing solvers. They are not amenable to partitioning because they rely on solvers that do not support partitioning. Examples include SATPLAN [9] that transforms a planning problem into a satisfiability (SAT) problem, Blackbox [10] that transforms a planning graph [1] into a SAT problem, and ILP-PLAN [11] that transforms a planning problem into an integer linear programming (ILP) problem with discrete metric constraints and optimization objectives.

In this paper, we formulate in Section 2 a planning problem as a discrete constrained optimization problem. Although this is not a customary representation for planning problems, it is needed in developing a formal mathematical foundation for resolving global constraints. We present in Section 3 our theory of extended saddle-point condition (ESPC) in discrete space and its decomposition into partitioned conditions. We then describe our implementation of the partitioned conditions in Section 4 and their application on improving ASPEN in Section 5. Finally, conclusions are drawn in Section 6.

2 Mathematical Formulation of Discrete Constrained Optimization

Our formulation assumes that the discrete horizon is partitioned in $N+1$ stages, with $u_t$ local variables, $m_t$ local equality constraints, and $r_t$ local inequality constraints in stage $t$, $t = 0, \ldots, N$. Such partitioning decomposes the discrete variable vector $y \in D^w$ of the problem into $N+1$ subvectors $y(0), \ldots, y(N)$, where $y(t) = (y_t(0), y_t(1), \ldots, y_t(V_t))$ is the $t$th element state vector in discrete space at stage $t$, and $y_t(t)$ is the $t$th dynamic state variable in stage $t$. A solution to such a problem is a plan that consists of the assignments of all variables in $y$. A single-objective formulation of the problem is as follows:

$$\begin{align*}
(P_2) \quad \min_y & \quad J(y) \\
\text{subject to} & \quad h^0(y(t)) = 0, \quad g_t^0(y(t)) \leq 0, \quad t = 0, \ldots, N, \quad (\text{local constraints}), \\
& \quad H(y) = 0, \quad G(y) \leq 0, \quad (\text{global constraints}).
\end{align*}$$

Here, $h^0 = \{ h_0, \ldots, h_m \}^T$ and $g^0 = \{ g_1^0, \ldots, g_r^0 \}^T$ are local-constraint functions that involve $y(t)$ and time in stage $t$; and $H = [H_1, \ldots, H_N]^T$ and $G = [G_1, \ldots, G_N]^T$ are global-constraint functions that involve state variables and time in two or more stages. Note that constraint may involve conditions on individual states, preconditions on an action, conditions to be maintained throughout an action, and post-conditions to be achieved by an action, and that the functions are not required to be continuous and differentiable.

A planning problem may also involve the optimization of one or more objectives. An important property commonly considered necessary for any feasible candidate solution to a multi-objective optimization problem is Pareto optimality [20]. A Pareto optimal set consists of Pareto optimal solutions (POS) that are not dominated by any other solutions, where solution $y'$ dominates solution $y$ if $y'$ is worse than or equal to $y$ in all objectives, with at least one strictly worse. Most search algorithms look for one or more POS in the Pareto optimal set.

There are several approaches for finding POS in unconstrained space. Consider a problem of optimizing $J(y)$ that consists of a vector of $k$ objective functions:

$$\min_y J(y) = (J_1(y), J_2(y), \ldots, J_k(y))^T.$$

The first class of methods are those that transform the multi-objective functions into a single objective and try to find only one POS. The easiest and widely used approach is the weighted-sum method [20] that combines the multiple objectives linearly into a single objective using a vectors of weights, one for each objective. A new POS can be found by varying the weights and by solving the single-objective problem for each combination of weights. The main disadvantage of this method is that all POS in the Pareto optimal set can only be generated when all the objective functions are convex. In the special case of looking for local POS (with respect to other local POS in the neighborhood), the convexity assumption is satisfied when the objective functions are continuous and differentiable, then falls in general for discrete objective functions considered in this paper. In the latter, there may not exist weights for some local POS with respect to other local POS in their discrete neighborhoods.

The norm method is based on minimizing the relative distance from a candidate solution to an ideal reference solution vector $(\bar{J}_1, \ldots, \bar{J}_k)$. It transforms the multiple objectives into the following single objective with integer $p$:

$$\min_y J(y) = \left[ \sum_{i=1}^k w_i \left( \frac{J_i(y) - \bar{J}_i}{\bar{J}_i} \right)^p \right]^{1/p},$$

where each POS is associated with a fixed combination of weights. It represents a family of methods because different distance measures are obtained by varying $p$, and more POS
are expected to be found in nonconvex problems using a larger \( p \). However, for finite \( p \), the method cannot guarantee that all \( P \) are found, even for all possible combinations of weights, unless its objectives are convex.

The minimax method [20], [7] can potentially generate all \( P \) for nonconvex problems by minimizing the maximum of the weighted criteria in the feasible set, leading to a scalar objective at point \( y \) as follows:

\[
\min_y \ J(y) = \frac{1}{n} \sum_{i=1}^n u_i J_i(y).
\]

This is a special case of (3) in which \( p = \infty \) and \( J^*_i = 0 \). In contrast to norm methods using finite \( p \), only the minimax approach guarantees that all \( P \) are feasible. We adopt this approach in (1) because our criterion for selecting a suitable objective in an optimization is that a systematic search can lead to all \( P \) in the Pareto optimal set.

3 Extended Saddle-Point Conditions in Discrete Space

Consider the following discrete optimization problem \( P_d \):

\[
(P_d) : \quad \min_y \ f(y), \ y \in D^m \ \\
\text{subject to} \quad h(y) = 0 \text{ and } g(y) \leq 0.
\]

The goal of solving \( P_d \) is to find a constrained local minimum \( y \) with respect to \( \mathcal{N}_d(y) \), the discrete neighborhood of \( y \).

**Definition 1.** A user-defined discrete neighborhood \( \mathcal{N}_d(y) \) of \( y \in D^m \), is a finite user-defined set of states \( \{ y \in D^m \} \) such that \( y' \in \mathcal{N}_d(y) \iff y \in \mathcal{N}_d(y') \), and that it is possible to reach every \( y' \) from any \( y \) in one or more steps through neighboring points.

Intuitively, \( \mathcal{N}_d(y) \) represents points that can be reached from \( y \) in one step, regardless of whether there is a valid action to effect the transition.

**Definition 2.** Point \( y' \) is a constrained local minimum in the discrete neighborhood (CLMd) of \( P_d \) if \( y' \) is feasible and \( f(y') \leq f(y) \) for all feasible \( y \in \mathcal{N}_d(y') \).

There are two distinct features of CLMd. First, the set of CLMd of a problem is neighborhood-dependent because it depends on the user-defined discrete neighborhood; that is, it may be CLMd with respect to \( \mathcal{N}_d(y) \) but may not be with respect to \( \mathcal{N}_d(y') \). Although the choice of neighborhoods does not affect the validity of a search as long as a consistent definition is used throughout, it may affect the time to find a CLMd. Second, a discrete neighborhood has a finite number of points. Hence, the verification of \( y \) to be CLMd with respect to \( \mathcal{N}_d(y) \) can be done by comparing its objective value against that of its finite number of discrete neighborhoods. This feature allows the search of a descent direction in discrete neighborhood to be done by enumeration or greedy search, rather than by differentiation.

Before we state the main theorem, we define a new Lagrangian function in discrete space:

**Definition 3.** The \( \ell_1 \)-penalty function of \( P_d \) in (5) is defined as follows:

\[
L_d(y; \alpha, \beta) = f(y) + \alpha^T [h(y)] + \beta^T \max(0, g(y)),
\]

where \( \alpha \) and \( \beta \) are the extended Lagrange multipliers.

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**Theorem 1.** Necessary and sufficient extended saddle point condition (ESPC) on CLMd of \( P_d \) [21] Suppose \( y' \in D^m \) is a point in the discrete space of \( P_d \). Then \( y' \) is a CLM of \( P_d \) iff there exist finite \( \alpha^* \geq 0 \) and \( \beta^* \geq 0 \) such that, for any \( \alpha > \alpha^* \) and \( \beta > \beta^* \), the following saddle-point condition is satisfied:

\[
L_d(y; \alpha, \beta) \leq L_d(y^*; \alpha^*, \beta^*) \leq L_d(y^*; \alpha^*, \beta^*)
\]

for all \( y \in \mathcal{N}_d(y^*) \), \( \alpha \in \mathbb{R}^n \), and \( \beta \in \mathbb{R}^r \).

The proof of the theorem can be found in the references [23], [21].

The inequalities in (7) state that \( \{ y^*, \alpha^*, \beta^* \} \) is at a local minimum of \( L_d(y; \alpha, \beta) \) with respect to \( y \) and \( (\alpha, \beta) \) with \( y \) and \( \alpha \) and \( \beta \).

Based on Theorem 1, we can now solve \( P_d \) in (1) by partitioning it into subproblems. We first show that plan \( y^* \), a CLM of \( P_d \) with respect to its discrete neighborhood \( \mathcal{N}_d(y^*) \), satisfies the ESPC in Theorem 1. To solve (1) efficiently, we define a separable neighborhood and partition the ESPC in (7) into a set of necessary conditions that collectively are necessary and sufficient. The partitioned conditions can then be implemented by finding local saddle points in each stage of \( P_d \) and by resolving the unsatisfied global constraints using appropriate Lagrange multipliers.

To enable the partitioning of ESPC into independent necessary conditions, we define the separable neighborhood of plan \( y \) as follows:

**Definition 4.** Given \( \mathcal{N}'_d(y(t)) \), the discrete neighborhood of \( y(t) \) stage \( t \), we define \( \mathcal{N}'_d(y) \), the separable discrete neighborhood of \( y \) as follows:

\[
\mathcal{N}_d(y) = \bigcup_{t=0}^N \mathcal{N}'_d(y(t)) = \bigcup_{t=0}^N \left\{ y_{i(t)} \in \mathcal{N}'_d(y(t)) \text{ and } y_{i(t)}(t) = y(t) \right\}.
\]

Intuitively, \( \mathcal{N}_d(y) \) is partitioned into \( N + 1 \) neighborhoods, each partitioning \( y \) into one of the stages of \( P_d \). By considering \( P_d \) in (1) as a discrete optimization problem, we can apply (6) and Theorem 1 to get the ESPC condition. Based on the separable neighborhood, our main theorem shows the partitioning of this condition into a set of partitioned conditions.

**Definition 5.** The \( \ell_1 \)-penalty function of \( P_d \) in (5) is defined as follows:

\[
L_d(y; \alpha, \beta, \gamma, \eta) = f(y) + \gamma^T |h(y)| + \eta^T \max(0, g(y)) \]

\[
+ \gamma^T |H(y)| + \eta^T \max(0, G(y)),
\]

where \( \gamma = (\gamma_1, \ldots, \gamma_n) \in \mathbb{R}^m \), \( \beta = (\beta_1, \ldots, \beta_n) \in \mathbb{R}^n \), \( \gamma = (\gamma_1, \ldots, \gamma_n) \) and \( \eta = (\eta_1, \ldots, \eta_n) \) \in \mathbb{R}^n \) are vectors of extended Lagrange multipliers.

Next, we show that (7) can be partitioned into a set of necessary and sufficient conditions.

**Theorem 2.** Partitioned necessary and sufficient ESPC on CLMd of \( P_d \). Plan \( y \) is a CLM of \( (1) \) with respect to \( \mathcal{N}_d(y) \) iff the following \( N + 2 \) conditions are satisfied:

\[
\Gamma_d(y^*; \alpha, \beta, \gamma^*, \eta^*) \leq \Gamma_d(y^*; \alpha^*, \beta^*, \gamma^*, \eta^*) \leq \Gamma_d(y^*; \alpha^*, \beta^*, \gamma^*, \eta^*)
\]

\[
L_d(y, \alpha^*, \beta^*, \gamma^*, \eta^*) \leq L_d(y, \alpha^*, \beta^*, \gamma^*, \eta^*)
\]
for all \( y \in N_{g_{0}}^{0}(y^{*}) \) and \( \alpha(t) \in \mathbb{R}^{n}, \beta(t) \in \mathbb{R}^{r}, \gamma \in \mathbb{R}^{r}, \eta \in \mathbb{R}^{r}, \) and \( t = 0, \ldots, N, \) where
\[
\Gamma_{d}^{0}(y, \alpha(t), \beta(t), \gamma, \eta) = J(y) + \mathbf{v}^{T}H(y(t)) + \beta(t)^{T} \max(0, g_{0}^{0}(y(t))) + \gamma^{T}H(y) + \eta^{T} \max(0, G(y)).
\]

Proof. We prove the theorem by showing the equivalence of ESPC in (7) and those in (10) and (11).

1st step: Given \( y^{*} \) that satisfies (7), we show that it also satisfies (10) and (11). Since for all \( t = 0, \ldots, N, \) any \( y \in N_{g_{0}}^{0}(y^{*}) \) is also a point in \( N_{g}(y^{*}) \); hence, the inequality on the right of (10) is implied by the inequality on the right of (7). The inequality on the left of (10) and that in (11) are obvious, as all the constraints are satisfied at \( y^{*} \).

2nd step: We prove this part by contradiction. Assuming that \( y^{*} \) satisfies (10) and (11) but not (7), the inequality on the left of (7) cannot be violated because the inequalities on the left of (10) and (11) imply that all local and global constraints are satisfied. Therefore, it must be the inequality on the right of (7) that is not satisfied at \( y^{*} \). That is, there exist \( y \in N_{g}(y^{*}) \) and a unique \( t' \) where \( y \in N_{g_{0}}^{0}(y^{*}) \) (according to the definition of \( N_{g}(y) \) in (8)) such that:
\[
L_{d}(y^{*}, \alpha^{*}, \beta^{*}, \gamma^{*}, \eta^{*}) \geq L_{d}(y, \alpha^{*}, \beta^{*}, \gamma^{*}, \eta^{*}).
\]

(12)

This implies that the inequality on the right of (10) is not satisfied at \( t = t' \), which contradicts our assumption that \( y^{*} \) satisfies (10) and (11). Note that \( t' \) exists because the neighborhood in (8) is separable. These two parts prove the correctness of the theorem.

By using a separable neighborhood, Theorem 2 shows that the original ESPC in Theorem 1 can be partitioned into \( N + 1 \) necessary conditions, each of which corresponds to finding a saddle point in a stage of the original problem. Hence, the original problem is now reduced to solving multiple smaller subproblems and to the resolution of unsatisfied global constraints across the stages. By reducing the solution space in each subproblem through the search of extended saddle points, Theorem 2 leads to a significant reduction in the base of the exponential complexity in finding \( CL_{M_{d}} \).

4 Global Search Implementing ESPC in Discrete Space

An important aspect of Theorem 1 is that it suffices to find any \( \alpha^{*} > \alpha^{0} \) and \( \beta^{*} > \beta^{0} \) in order to satisfy the ESPC. Such a property allows the solution of \( P_{d} \) to be found iteratively by looking for a local minimum \( y^{*} \) of \( L_{d}(y, \alpha, \beta) \) with respect to points in \( N_{g}(y^{*}) \) in an inner loop, and for any \( \alpha^{*} > \alpha^{0} \) and \( \beta^{*} > \beta^{0} \) in an outer loop.

Figure 4a shows the pseudo code that implements the conditions in Theorem 1. The inner loop looks for local minima of \( L_{d}(y, \alpha, \beta) \) in its discrete neighborhoods with respect to \( y \), whereas the outer loop performs ascents on \( \alpha \) and \( \beta \) for unsatisfied global constraints and stops when a \( CL_{M_{d}} \) has been found.

The iterative search can be extended to the conditions in Theorem 2. In Figure 4b, the two inner nested loops of stage \( t \) look for a local saddle point of \( \Gamma_{d}^{0}(y, \alpha(t), \beta(t), \gamma, \eta) \) This is done by updating \( y, \alpha(t), \) and \( \beta(t) \) associated with the local constraints, using fixed \( \gamma \) and \( \eta \) associated with the global constraints. With fixed \( \gamma \) and \( \eta \), the algorithm is actually finding \( y(t) \) that solves the following discrete optimization problem in stage \( t \):}

\[
\min_{y(t)} J(y) + \gamma^{T}H(y) + \eta^{T}G(y)
\]

subject to \( h_{0}(y(t)) = 0 \) and \( g_{0}^{0}(y(t)) \leq 0 \).

Since this is a well-defined optimization problem, any existing solver with little modification can be used to solve it. In the next section, we apply ASPEN to solve partitioned planning subproblems in the form of (13).

After performing the local searches, the penalties on unsatisfied global constraints are increased in the outer loop. The search iterates until a feasible local minimum in the constrained model has been found.

A search using the above simple implementations may get stuck in an infeasible region when the objective is too small or when the Lagrange multipliers and/or constraint violations are too large. In this case, increasing the Lagrange multipliers will further deepen the infeasible region, making it impossible for a local-descent algorithm in the inner loops to escape from this region.

To address this issue, we can change either the ascent algorithm in the two outer loops of Figure 4b or its descent algorithm in the innermost loops. The ascent algorithm can be changed to increase as well as decreases of Lagrange multipliers \( \alpha, \beta, \gamma, \) and \( \eta \). The goal of decreases is to "lower" the barrier in the Lagrangian function in order for local descents in the innermost loops to escape from an infeasible region. Note that \( \alpha, \beta, \gamma, \) and \( \eta \) should be decreased gradually in order to help the search escape from local minima of \( L_{d}(y, \alpha, \beta, \gamma, \eta) \). Once \( \alpha, \beta, \gamma, \) and \( \eta \) reach their maximum thresholds, they can be scaled down and the search repeated. In our partitioned implementation of ASPEN described in the next section, we have set upper bounds on Lagrange multipliers and scale the multipliers once they reach the upper bounds. However, we have found that periodic decreases of the multipliers before they reach the upper bounds is not necessary because the solution space has many feasible solutions and the search never gets stuck in an infeasible region.

In a similar way, the descent algorithm in the innermost loops can be changed to allow descents as well as ascents. In temporal planning problems, the exact gradient direction of functions may not be available because the functions are not in closed form. As a result, a randomly generated probe does not look like descent directions, and a deterministic descent procedure may get stuck easily in infeasible local minima. To cope with this issue, probes generated may be accepted based on random criteria, or diagonal search and descent schemes as well as ascents. In the next section, we describe our partitioned implementation of ASPEN [3] that accepts probes with larger Lagrangian values according to the Metropolis probability.
5 Experimental Results on a Partitioned Implementation of ASPEN

In this section we show experimental results on implementing our partitioned search in Theorem 2 in ASPEN.

ASPEN alternates between a repair phase and an optimization phase because it cannot optimize plan quality and search for feasible plans at the same time. In the repair phase [17], ASPEN generates a random plan that may have conflicts and searches for a feasible plan from this initial plan, using iterative repairs to resolve conflicts. In a repair step, the planner must decide at each choice point a conflict to resolve and a conflict-resolution method from a rich collection of repair heuristics. Next, in the optimization phase, ASPEN uses a preference-driven, incremental, local optimization to optimize plan quality defined by a preference score. It decides the best search direction at each choice point, based on information from multiple choice points. In our experiments, we allow ASPEN to alternate between a repair phase with an unlimited number of iterations and an optimization phase with 200 iterations.

We have compared the performance of the various implementations using OPTIMIZE, PREP, DCAPS, and CXI-PREP. These four publicly available benchmarks on scheduling parallel spacecraft operations encode goal-level tasks commanded by science and engineering operations personnel, with a goal of generating high-quality plans as fast as possible. OPTIMIZE (10 objectives) and PREP (50 objectives) are two benchmarks developed at JPL that come with the licensed release of ASPEN. The CXI-PREP benchmark [22] (7 objectives) models the operations planning of the Citizen-Explorer-1 (CX-1) satellite that took data relating to ozone and downlinked its data to ground for scientific analysis. It has a problem generator that can generate problem instances of different number of satellite orbits. Last, the DCAPS benchmark [16], managed by the University of Colorado at Boulder, models the operation of DATA-CHASER shuttle payload. Since it has no preferences, we define a preference of one for a feasible solution and zero otherwise.

Figure 5 shows ASPEN+PART(N, PARTITION.STRATEGY), a partitioned implementation of ASPEN for solving planning problems in N stages. Here, we set a weight of 100 in both the single and minmax objective (4) in the Lagrangian function (since the preference score is between 0 to 1), and initialize all Lagrangian multipliers to zeros.

In generating a new plan from the current plan during descents of $I_d^*(t)$ (Line 8 in Figure 5), ASPEN chooses probabilistically among its repair and optimization actions, selects a random feasible action at each choice point, and applies the selected actions to the current plan.

As is discussed in the last section, since many of the objectives and constraints in complex spacecraft applications are not differentiable, a new plan generated does not likely follow descent directions, and a local descent of the Lagrangian function may get stuck easily in infeasible local minima. To address this issue, ASPEN+PART(N, PARTITION.STRATEGY) employs the Metropolis probability $A_T$ to determine whether to accept a new plan (Line 9 in Figure 5). Using a parameter called temperature $T$, it accepts the new plan with larger $I_d^*(t)$ based on the following $A_T$, with the acceptance probability decreasing as $T$ decreases:

$$A_T(y, y') = \exp\left(-\frac{(L_d(y') - L_d(y))^+}{T}\right),$$

where $y = (y, \alpha, \beta, \gamma; \eta)$; $y' = (y', \alpha, \beta, \gamma; \eta)$; and $(a)^+ = a$ if $a > 0$ and $(a)^+ = 0$ otherwise for all $a \in \mathbb{R}$. We have used a geometric cooling schedule $T_{new} = c \cdot T_{old}$ because the logarithmic cooling schedule is too slow. In our experiments, $c = 0.8$ and the initial temperature is 1,000.

The stochastic approach proposed above requires the modified planner to generate a candidate plan, determine whether to accept it based on the Metropolis probability, and reverse the changes made when the candidate is not accepted. However, the reversal of changes is difficult in ASPEN because ASPEN commits to every repair/optimization action it generates. To address this issue, we create a child process that is a duplicate of the ASPEN program in memory before we evaluate a candidate plan. We apply the scheduling actions on the child copy, evaluate the plan, and carry out the same actions in the main process only if the new plan is accepted. Although the CPU overhead of forking a child process is significant and amounts to 10-20 times longer than the time of a normal iteration, this overhead will be marginal with an efficient implementation of undo in ASPEN. Moreover, since we measure the number of iterations according to the number of plans evaluated, including discarded ones, our results will accurately reflect the overhead in ASPEN if the undo operation were actually implemented.

The Lagrangian multipliers are updated in each iteration in Line 12 of Figure 5 by increasing the multipliers of unsatisfied constraints by 0.1. As is discussed in the last section, we set a maximum threshold of 1000 for each Lagrange multiplier and divide all of them by 1000 when one of them reaches the maximum.

Two other important issues that must be addressed in our partitioned implementation are the number of stages used and the duration of each. In ASPEN, a conflict has an active window bounded by a start time and an end time called the time points. Adjacent time points can be collapsed into a stage, since ASPEN has discrete time horizons.

We have studied both the static and dynamic partitioning of stages. In static partitioning, ASPEN+PART(N, STATIC) partitions the horizon statically and evenly into N stages. This simple strategy often leads to an unbalanced number of time points in different stages. During
Figure 6: Number of iterations taken by static and dynamic partitioning in ASPEN-PART(\(N\), PARTITION, STRATEGY) to find a feasible plan for the 8-orbit CX1-PREF problem.

a search, some stages may contain no conflicts to be resolved, whereas others may contain too many conflicts. Such an imbalance leads to search spaces of different sizes across different stages and search times that may be dominated by those in a few stages.

To achieve a better balance of activities across stages, ASPEN-PART(\(N\), DYN) adjusts the boundary of stages dynamically. This is accomplished by finding \(M\), the number of time points in the horizon related to conflicts, at the end of the outer loop (Line 15 in Figure 5) and by partitioning the horizon into \(N\) stages in such a way that each stage contains approximately the same number (\(M/N\)) of such time points (Line 19 in Figure 5). To determine the best \(N\), Figure 6 plots the number of iterations taken by static and dynamic partitioning in finding a feasible plan of the 8-orbit CX1-PREF problem. The results show that \(N = 100\) is optimal, although the performance is not very sensitive to \(N\) when it is larger than 100. Since other benchmarks lead to similar conclusions, we set \(N = 100\) in the following experiments.

Figure 7 compares the performance of the original ASPEN, ASPEN-PART(1, STATIC), ASPEN-PART(100, STATIC), and ASPEN-PART(100, DYN) on the five benchmarks described earlier, based on a single-objective formulation. In each graph, we plot the quality of the best feasible plan found with respect to the number of search iterations. The results show that our partitioned implementations are able to find plans of the same quality one to two orders faster than ASPEN and ASPEN-PART(1, STATIC) and much better plans when they converge. Further, dynamic partitioning can find better plans in shorter times than those found by static partitioning.

Figure 8 compares the performance of ASPEN and ASPEN-PART(100, DYN), where the latter is based on a multi-objective formulation. We plot ten different POS found by ASPEN-PART(100, DYN), using random sets of weights between 0 and 100 in the minimax formulation (4). In each case, we show the Euclidean distance between the objective vectors of the plan found to the Utopian objective vector in which all objective functions are of the maximum value 1.0. The results show that ASPEN-PART(100, DYN) can find a POS one to two orders faster than ASPEN and can generate multiple POS.

Table 1 further illustrates the various seven-objective solutions found on the 8-orbit CX1-PREF problem. It can be verified that, due to the existence of S3 and S10, there exists no combination of weights that makes S2 a global minimum in the weighted-sum objective used by ASPEN. This is true because, in order for S2 to be better than S10 on the weighted sum, the weight on J3 must be at least 155 times larger than that on J5; however, this will make S2 worse than S3 in terms of the weighted sum. As a result, it is not possible for ASPEN to find S2 using an objective based on a weighted sum.

Figure 7: Quality-time comparisons of ASPEN, ASPEN-PART(1, STATIC), ASPEN-PART(100, STATIC), and ASPEN-PART(100, DYN). (All runs involving ASPEN-PART were terminated at 24,000 iterations and used the Metropolis probability to accept probes with worse Lagrangian value during descents. The preference score of DCAPS is the same when a feasible solution is found and zero otherwise.)

6 Conclusions

In this paper, we have presented the extended saddle point condition in discrete space and its decomposition into partitioned conditions that collectively are necessary and sufficient for any constrained local minimum in discrete constrained optimization. The theory leads to an efficient iterative scheme for resolving global constraints across partitioned subproblems and for finding partitioned saddle points in each partitioned subproblem. We apply the partitioned conditions on the discrete-space ASPEN planner for solving partitioned planning benchmarks and demonstrate significant improvements, both in terms of the quality of the plans generated and the execution times to find them.
The partitioning approach presented is important for reducing the exponential complexity of nonlinear constrained optimization problems. By partitioning a problem into subproblems and by reducing the search space of each partitioned subproblem using our proposed theory, we can reduce the base the exponential complexity of the overall problem. Further, since variable partitioning leads to planning subproblems of similar nature but of small scale, we can exploit existing planners and their efficient pruning techniques to further reduce the search space of these subproblems.

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References


