QMF FILTER BANK DESIGN BY A NEW GLOBAL OPTIMIZATION METHOD

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ABSTRACT

In this paper, we study various global optimization methods for designing QMF (quadrature mirror filter) filter banks. We formulate the design problem as a nonlinear constrained optimization problem, using the reconstruction error as the objective, and other performance metrics as constraints. This formulation allows us to search for designs that improve over the best existing designs. We present NOVEL, a global optimization method for solving nonlinear continuous constrained optimization problems. We show that NOVEL finds better designs with respect to simulated annealing and genetic algorithms in solving QMF benchmark design problems. We also show that relaxing the constraints on transition bandwidth and stopband energy leads to significant improvements in the other performance measures.

1. INTRODUCTION

Digital filter banks have been applied in many engineering fields. Their design objectives consist of their overall performance and the performance of each individual filter. Figure 1 summarizes the various design objectives for measuring design quality. In general, filter bank-design problems are multi-objective, continuous, nonlinear optimization problems.

Algorithms for designing filter banks can be classified into optimization-based and non-optimization-based. In optimization-based methods, a design problem is formulated as a multi-objective nonlinear optimization problem [4] whose form may be application- and filter-dependent. The problem is then converted into a single-objective optimization problem and solved by existing optimization methods, such as gradient-descent, Lagrange-multiplier, quasi-Newton, simulated-annealing, and genetics-based methods. On the other hand, filter-bank design problems have also been solved by non-optimization-based algorithms, which include spectral factorization and heuristic methods. These methods generally do not continue to find better designs once a suboptimal design has been found.

Filter	Design Objectives
Overall Filter Bank	Minimize amplitude distortion
	Minimize aliasing distortion
	Minimize phase distortion
Single Filter	Minimize stopband ripple (δ_s)
	Minimize passpand ripple (δ_p)
	Minimize transition bandwidth (T_t)
	Minimize stopband energy (E_s)
	Maximize passband flatness (E_p)

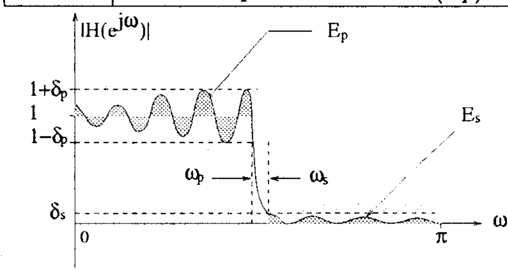

Figure 1: Possible design objectives of filter banks and an illustration of the design objectives of a single low-pass filter. ($[0, \omega_p]$ is the pass band; $[\omega_s, \pi]$, the stop band; $[\omega_p, \omega_s]$, the transition band.)

In this paper, we study global optimization methods for designing QMF FIR filter banks. These filter banks are an important class of filter banks that have been studied extensively. In a two-band QMF filter bank, the reconstructed signal is [1]:

$$\begin{aligned} \hat{X}(z) &= \frac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]\, X(z) \qquad (1) \\ &+ \frac{1}{2}[H_0(-z)F_0(z) + H_1(-z)F_1(z)]\, X(-z). \end{aligned}$$

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where $X(z)$ is the original signal, and $H_i(z)$ and $F_i(z)$ are, respectively, the response of the analysis and synthesis filters. To perfectly reconstruct the original signal based on $\hat{X}$, we have to eliminate aliasing, amplitude, and phase distortions. QMF FIR filter banks implement perfect reconstruction by setting $F_0(z) = H_1(-z)$, $F_1(z) = -H_0(-z)$ and $H_1(z) = H_0(-z)$, leading to one prototype filter $H_0(z)$ in the system, linear phase, and zero aliasing and phase distortions.

2. OPTIMIZATION DESIGN OF QMF FILTER BANKS

The design of QMF filter banks can be formulated as a multi-objective unconstrained optimization or as a single-objective constrained optimization.

In a multi-objective formulation, the goals can be to (a) minimize the amplitude distortion (reconstruction error) of the overall filter bank, and (b) maximize the performance of the individual prototype filter $H_0(z)$. A possible formulation is to optimize the design with respect to a subset of the measures defined in Figure 1.

$$Min \quad E_r \text{ and } E_s \tag{2}$$

Unfortunately, optimal solutions to the simplified optimization problem are not necessarily optimal solutions to the original problem. Oftentimes, performance measures not included in the formulation are compromised.

In general, optimal solutions of a multi-objective problem form a *Pareto optimal frontier* such that one solution on this frontier is not dominated by another. One approach to find a point on the Pareto frontier is to optimize a weighted sum of all the objectives. This approach has difficulty when Pareto frontier points of certain characteristics are desired, such as those with certain transition bandwidth. Different combinations of weights must be tested by trial and error until a desired filter is found. When the desired characteristics are difficult to satisfy, trial and error is not effective in finding feasible designs. In this case, constrained formulation should be used instead.

2.1. Single-Objective Constrained Formulation

In this formulation, constraints are defined with respect to a reference design. Constraint-based methods have been applied to design QMF filter banks in both the frequency and the time domains. In the frequency domain, the most often considered objectives are the reconstruction error, E_r, and the stopband ripple. As stopband ripples cannot be formulated in closed form, stopband attenuation is used instead (represented as E_s in (2)). In the time domain, Nayebi gave a time-domain formulation with constraints in the frequency domain and designed filter banks using an iterative time-domain design algorithm.

In this paper, we formulate the design of a QMF filter bank in the most general form as a constrained nonlinear optimization problem as follows.

$$\begin{aligned} &Minimize && E_r && \\ &\text{subject to} && E_p \le \theta_{E_p} && E_s \le \theta_{E_s} \\ &&& \delta_p \le \theta_{\delta_p} && \delta_s \le \theta_{\delta_s} \\ &&& T_t \le \theta_{T_t} && \end{aligned} \tag{3}$$

where θ_{E_p}, θ_{E_s}, θ_{δ_p}, θ_{δ_s} and θ_{T_t} are constraint bounds obtained in the best known design (with possibly some constraint values relaxed or tightened in order to obtain designs of different trade-offs). The goal here is to find designs whose performance measures are better than or equal to those of the reference design. Since the objective and constraints are nonlinear, the problem is multi-modal with many local minima.

2.2. Nonlinear Optimization Methods

Finding global optimal solutions of nonlinear continuous constrained problems is one of the most challenging tasks in optimization. There are two distinct strategies to handle constraints. (a) Absorb all constraints into the objective and weigh them by penalty terms. This is not effective because it is hard to choose appropriate penalty terms when constraints are violated. (b) Absorb constraints into a Lagrangian function, which is the sum of the objective and the constraints weighted by Lagrange multipliers.

In a Lagrangian formulation, a local minimum in a feasible region is a *saddle point* at which the objective function is at a local minimum and the weighted sum of the constraints is at a local maximum. By using this property, saddle points can be found by *local search methods* that perform gradient descents in the original-variable space and gradient ascents in the Lagrange-variable space.

Since local search methods converge to local minima, *global search methods* are needed to bring the search out of local minima. There are two classes of global search methods. (a) Deterministic methods, such as covering methods and generalized descent methods, do not work well when the search space is large. (b) Probabilistic methods are weak in either their local or global search. For instance, gradient information is not used well in simulated annealing and evolutionary algorithms. In contrast, gradient descent algorithms with multistarts and random probing are weak in their global search strategy.

2.3. NOVEL Global Optimization Method

In this subsection, we describe NOVEL (*Nonlinear Optimization via External Lead*) [3], a global optimization method that relies on an external force to pull the search out of local minima. Starting from a Lagrangian formulation, our implementation has two stages.

In the *global-search stage*, NOVEL looks for good starting points for the local-search stage. This is important because it first identifies good starting points before applying expensive local searches. This avoids repeatedly determining unpromising local minima as in multi-start algorithms and applying computationally expensive descent algorithms from random starting points. The result of this stage is a trajectory on the Lagrangian-function space. The dynamics of the trajectory is controlled by two forces: local gradient to pull the trajectory towards a local minimum, and the force exerted by a gradient-independent trace function to pull the trajectory out of a local minimum. The latter is particularly important because it provides a continuous means of going from one local region to another, avoiding problems in methods that determine new starting points heuristically and losing valuable local information found in a local search.

In the *local-search stage*, NOVEL uses promising starting points identified in the global-search stage and applies local searches to find saddle points in the Lagrangian function space. These local searches include gradient descents in the original-variable space and gradient ascents in the Lagrange-variable space. The designs found correspond to designs whose constraints are satisfied and whose objective is at a local minimum.

3. EXPERIMENTAL RESULTS

We have applied NOVEL to solve some QMF filter-bank design problems formulated by Johnston [2]. In our designs, we have used (3) with constraint bounds defined by those of Johnston's designs. Our goal is to find designs that are better than Johnston's results across all six performance measures, as well as those when one or a few constraints are relaxed.

Table 1 compares the performance of 24D, 32D and 48D QMF filter-bank designs obtained by NOVEL with respect to Johnston's [2]. Our objective is to minimize E_r with other measures of Johnston's as constraints. In all three cases, our designs have smaller reconstruction errors and passband energies, while all other measures are either better than or equal to those of Johnston's [2].

Johnston used sampling in computing energy values whereas NOVEL used closed-form integration. Hence, designs found by Johnston are not necessarily at the local minima in a continuous formulation. To demonstrate this, we applied local search in a continuous formulation of the 24D design, starting from Johnston's design. We found a design with a reconstruction error of 3.83E-05, which is better than Johnston's result of 4.86E-05. By applying global search, NOVEL can further improve the design to result in a reconstruction error of 3.66E-05.

We have applied simulated annealing (SA) and evolutionary algorithms (EA) in QMF filter-bank design. The SA we have used is Simann from netlib that works on a weighted-sum formulation. The EA is Sprave's Lice (Linear Cellular Evolution) that can be applied to both constrained and weighted-sum formulations. We have tried various parameter settings and report the best solutions in Table 2.

Table 1: Performance of 3 QMF filters obtained by NOVEL normalized with respect to Johnston's designs [2]. The objective is to minimize E_r with other measureas of Johnston's as constraints.

Performance	24D	32D	48D
E_r	0.75	0.87	0.95
E_p	0.77	0.80	0.76
E_s	1.00	1.00	1.00
δ_p	1.00	1.00	1.00
δ_s	1.00	1.00	1.00
$\Delta\omega$	1.00	1.00	1.00

Table 2: Comparing NOVEL, simulated annealing (SA) and evolutionary algorithm (EA) in designing 32-tap QMF filters. Measurements are normalized with respect to corresponding values of Johnston's designs [2].

	NOVEL	SA	EA-Constr	EA-Wt
32 c				
E_r	0.959	0.959	31.32	0.985
E_p	0.748	0.748	7.667e7	0.816
E_s	1.000	1.000	8.015e5	1.000
δ_p	1.000	1.000	3899	0.999
δ_s	1.000	1.000	401.9	0.844
$\Delta\omega$	1.000	1.000	0.000	1.001
32 d				
E_r	0.870	0.617	5.315	0.526
E_p	0.802	0.570	6.975e6	0.359
E_s	1.000	1.000	2.112e4	0.994
δ_p	1.000	0.769	1089	0.724
δ_s	1.000	1.000	678.0	1.000
$\Delta\omega$	1.000	1.015	0.000	1.042
32 e				
E_r	0.712	0.500	0.0943	0.507
E_p	0.896	0.582	1.542e5	0.590
E_s	1.000	1.000	1262	0.999
δ_p	1.000	1.000	1698	0.997
δ_s	1.000	1.000	17.30	0.999
$\Delta\omega$	1.000	1.013	0.000	1.013

Table 2 shows the performance of NOVEL, SA and EA in designing 32-tap QMF filters. SA and EA-Wt use weighted-sum formulation with weight 1 for reconstruction error and weight 10 for the rest performance measures. EA-Constr works on the same constrained formulation as NOVEL does. All methods were run significantly long, over 10 hours on a SUN SPARC20 workstation for each run.

NOVEL improves Johnston's solutions constantly. SA gets the same result as NOVEL does for 32c filter. However for 32d and 32e filters, the solution of SA has larger transition bandwidth than Johnston's. EA-Wt also has difficult in improving over Johnston's solution across all performance measures. In particular, solutions of EA-Wt have larger transition bandwidth.

EA-Constr converges to solution with very small reconstruction errors, while constraints are violated significantly. This is because constraints based on Johnston's solutions form a tiny feasible region in the search space. Randomly generated points have little chance of being feasible. Infeasible solutions are ranked by objective values regardless degree of constraint violation.

To summarize, performance improvements in NOVEL come from three sources. First, the closed-form formulation used in NOVEL is more accurate than the sampling method used in Johnston's approach. Local optima found by NOVEL are true local optima. Second, NOVEL uses a constrained formulation which allows it to find designs that are guaranteed to be better than or equal to Johnston's design with respect to all performance measures. Third, NOVEL employs effective global optimization strategies that allows it to explore a large part of the search space without first committing to many expensive local searches.

By using our constrained formulation, we can further study trade-off in designing QMF filter banks in a controlled environment. Loosening constraints in (3) generally leads to smaller reconstruction error.

Figure 2 demonstrates these trade-offs for 32d QMF filter banks. In our experiments, we have used Johnston's designs as our baselines. In the upper graph, constraints on stopband ripple and energy are relaxed by 5% to 50% from Johnston's solution. In the lower graph, constraint on transition bandwidth is relaxed by 5% to 50% from Johnston's solution. The y-axis shows the solution ratio, which is the ratio between the measure found by NOVEL and that of Johnston's.

When constraints on stopband ripple and energy are loosened, reconstruction error, passband ripple and energy decrease, while transition bandwidth is the same with respect to different relaxation ratio. When constraints on transition bandwidth is loosened, reconstruction error, passband energy and decrease significantly. Stopband ripple is reduced slightly while stopband energy is the same.

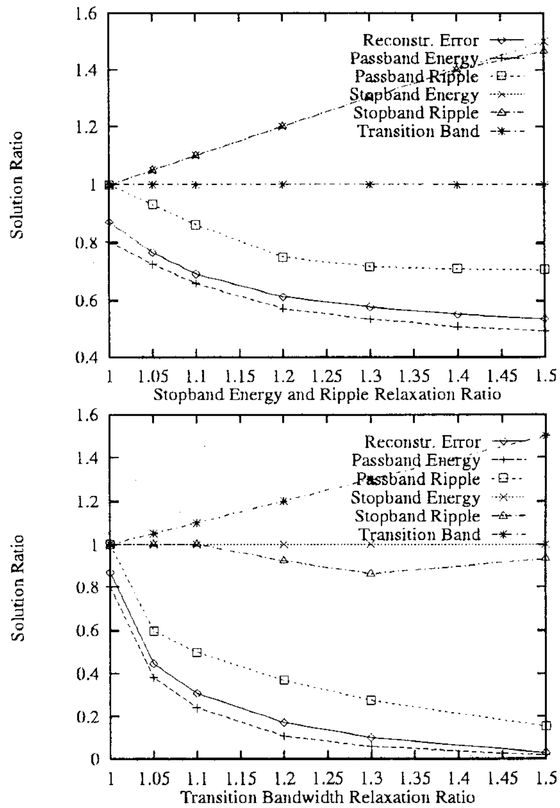

Figure 2: Experimental results in relaxing the constraints with respect to Johnston's designs for 32D QMFs. The x-axis shows the relaxation ratio of stopband energy and ripple (upper) , or transition bandwidth (lower) as constraints in NOVEL with respect to Johnston's value. The y-axis shows the ratio of the measure found by NOVEL with respect to Johnston's.

4. REFERENCES

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