CONSTRAINED OPTIMIZATION OF FILTER BANKS IN SUBBAND IMAGE CODING

Tao Wang and Benjamin W. Wah*
Department of Electrical and Computer Engineering
and the Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
Urbana, IL 61801, USA
email: {wangtao,wah}@manip.crhc.uiuc.edu
URL: http://manip.crhc.uiuc.edu

Abstract - The design of filter banks in subband image coding is critical for achieving high image quality. In this paper, we study the design from both the signal processing domain and the theory of wavelets. We formulate the design of filter banks as a two-stage nonlinear constrained optimization problem, each of which is solved by sequential quadratic programming (SQP). Using a wavelet image coding prototype, we show improved quality of the designed filter banks in terms of image-compression and peak signal-to-noise ratios (PSNRs).

INTRODUCTION

Subband image coding is a transform-based coding technique that uses a subband transform to reduce or remove spatial redundancies. This transform can be represented by a filter bank (Figure 1) with an analysis subsystem, including filters $H_0(z)$, $H_1(z)$, and two decimators, followed by a synthesis subsystem, including filters $F_0(z)$, $F_1(z)$, and two interpolators.

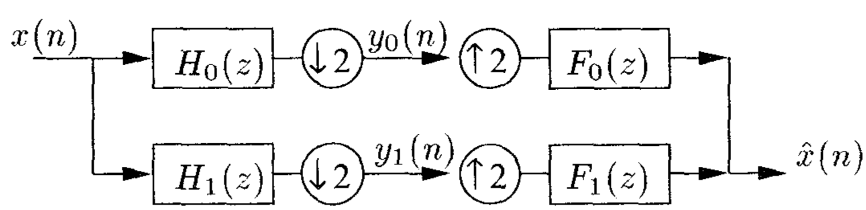

Figure 1: The structure of a two-band filter bank.

To achieve good quality in image coding, we need to select a proper filter-bank type and find good filter parameters [7, 1]. We select the type of filters

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based on two criteria: *perfect reconstruction* (PR) and *linear phase* (LP). The former precludes any errors caused by the filter bank itself, whereas the latter avoids introducing any phase errors into the transform coefficients, since phase distortions around edges are very visible.

A biorthogonal filter bank achieves both of these requirements. The filter bank requires only two filters $H_0(z)$ and $H_1(z)$ to be designed because its synthesis filters $F_0(z)$ and $F_1(z)$ are related to its analysis filters $H_0(z)$ and $H_1(z)$ by the relations $F_0(z) = 2H_1(-z)$ and $F_1(z) = -2H_0(-z)$. In addition, it is required [6] that the sum of filter lengths N_0 and N_1 be a multiple of 4, *i.e.*, $N_0+N_1 = 4m$, and that the filters satisfy one of the following conditions: a) both $H_0(z)$ and $H_1(z)$ are of even lengths, $H_0(z)$ is symmetric, and $H_1(z)$ is antisymmetric; or b) both filters are of odd lengths and are symmetric. In this paper we use filters of the second type.

FILTER DESIGN CRITERIA

Criteria for the Overall Filter Bank

Let $h_0(n), n = 1, 2, \cdots, N_0$ be the coefficients of low-pass filter $H_0(z)$, and $h_1(n), n = 1, 2, \cdots, N_1$ be those of high-pass filer $H_1(z)$. Since both filters are symmetric, LP is guaranteed. If the delay of the filter bank is $\frac{N_0+N_1}{2} - 1$, then PR can be enforced by a set of equations called the *PR condition*:

$$\frac{1}{2}\theta\left(i - \frac{N_0+N_1}{4}\right) = \sum_{k=1}^{2i}(-1)^{k-1}h_0(2i+1-k)h_1(k) \qquad (1)$$

for $i = 1, 2, \cdots, \frac{N_0+N_1}{4}$, where $\theta(x) = 1$ if $x = 0$, and 0 otherwise.

Criteria for Each Individual Filter

Using a filter bank, we split the original image into a set of subimages and quantize finer some subimages with lower frequencies than others with higher frequencies. If the subimages are separated well by the analysis filters, we will have good image coding quality; otherwise, some important information may leak into unimportant subimages, in which coarse quantization there may cause poor image coding quality.

To perfectly divide the frequency band into low and high frequencies, we require ideal filters. Since it is impossible to achieve ideal filtering using finite-length filters, we want to maximize the proximity of the designed filters to ideal filters. The degree of proximity can be simply measured by using passband energy E_p and stopband energy E_s for given passband and stopband cut-off frequencies ω_p and ω_s, respectively [4]. Let the Fourier transforms of filters h_0 and h_1 be, respectively, $FT_0(e^{j\omega}) = H_0(\omega)e^{-j\frac{N_0-1}{2}\omega}$ and $FT_1(e^{j\omega}) =$

$H_1(\omega)e^{-j\frac{N_1-1}{2}\omega}$, where

$$H_0(\omega) = h_0\left(\frac{N_0+1}{2}\right) + \sum_{n=1}^{(N_0-1)/2} 2h_0(n)cos\left(\frac{N_0+1}{2} - n\right)\omega \qquad (2)$$

$$H_1(\omega) = h_1\left(\frac{N_1+1}{2}\right) + \sum_{n=1}^{(N_1-1)/2} 2h_1(n)cos\left(\frac{N_1+1}{2} - n\right)\omega.$$

For given ω_s and ω_p, E_s and E_p for filters h_0 and h_1 are as follows:

$$E_s(h_0) = \int_{\omega_s}^{\pi} H_0^2(\omega)\,d\omega \qquad E_p(h_0) = \int_0^{\omega_p} (H_0(\omega)-1)^2\,d\omega \qquad (3)$$

$$E_s(h_1) = \int_0^{\pi-\omega_s} H_1^2(\omega)\,d\omega \qquad E_p(h_1) = \int_{\pi-\omega_p}^{\pi} (H_1(\omega)-1)^2\,d\omega.$$

Regularity Criteria

Derived from wavelet theory and also known as a *smoothness constraint*, regularity requires the iterated filters to converge to continuous functions [7]. For low-pass filter h_0, the regularity of order N requires at least N zeros in its amplitude response $H_0(\omega)$ at $\omega = \pi$:

$$H_0(\omega)|_{\omega=\pi} = 0 \qquad d^n H_0(\omega)/d\omega^n|_{\omega=\pi} = 0 \qquad (4)$$

for $n = 1, 2, \cdots, N-1$. Similar conditions can be defined for high-pass filter $H_1(\omega)$ except that it is evaluated at frequency $\omega = 0$.

Here we use 2-order regularity because it is sufficient for image coding [1]. The 2-order regularity can be enforced by having $H_0(\omega = \pi) = 0$ for low-pass filter h_0 and $H_1(\omega = 0) = 0$ for high-pass filter h_1:

$$h_0\left(\frac{N_0+1}{2}\right) + 2\sum_{n=1}^{(N_0-1)/2} (-1)^{(\frac{N_0+1}{2}-n)} h_0(n) = 0 \qquad (5)$$

$$h_1\left(\frac{N_1+1}{2}\right) + 2\sum_{n=1}^{(N_1-1)/2} h_1(n) = 0$$

Coding Gain

Coding gain measures energy compaction, and high coding gains correlate consistently with high objective values. By modeling a natural image as a Markovian source with the nearest sample correlation ρ and by assuming uncorrelated quantization errors, Katto and Yasuda [5] derived a filter-dependent coding gain:

$$G(\rho) = \frac{1}{\prod_{k=0}^{M-1}(A_k B_k)^{1/M}} \qquad (6)$$

where M is the number of subbands ($M = 2$), $A_k = \sum_i \sum_j h_k(i)h_k(j)\rho^{|j-i|}$, and $B_k = \sum_i f_k(i)^2$.

There are two ways to maximize coding gain [2]. First, it can be optimized in the first stage of subband decomposition with $\rho = 0.8$ and employed in other stages. Second, it can be optimized in every stage of subband transform with $\rho = 0.95$. Here we use the first strategy, because its computation is independent of subband division, leading to a filter bank that is applicable for any subband decomposition structures.

TWO-STAGE CONSTRAINED OPTIMIZATION

To design analysis filters h_0 and h_1, we have identified four sets of performance metrics formulated as a multi-objective problem as follows:

obj_1: Satisfy PR condition (1);

obj_2: Minimize $E_s(h_0)$, $E_s(h_1)$, $E_p(h_0)$, and $E_p(h_1)$ (3);

obj_3: Satisfy 2-order regularity (5);

obj_4: Maximize coding gain (6).

Since there is no multi-objective optimization method to solve this problem, we transform it into a constrained optimization problem using obj_1 and obj_4 as constraints. Note that it is hard to find the relative weights between obj_2 and obj_4 because they take values of different ranges.

Our solution strategy consists of two stages. First, we ignore obj_4 in our formulation and define the following constrained optimization problem:

$$\begin{array}{ll} \min_{h_0,h_1} & E_s(h_0) + E_p(h_0) + E_s(h_1) + E_p(h_1) \\ \text{subject to} & \text{PR condition (1)} \\ & 2 - \text{order regularity (5)} \end{array} \tag{7}$$

This is a nonlinear constrained optimization problem with a quadratic objective, two linear equality constraints, and $\frac{N_0+N_1}{4}$ quadratic equality constraints.

After obtaining a solution $\tilde{h}_0$ and $\tilde{h}_1$ to (7), we compute $\tilde{E}_{s0}$, $\tilde{E}_{p0}$, $\tilde{E}_{s1}$, and $\tilde{E}_{p1}$. Using them as performance bounds, we define the second optimization problem as follows:

$$\begin{array}{ll} \max_{h_0,h_1} & G(\rho) \\ \text{subject to} & \text{PR condition (1)} \\ & 2 - \text{order regularity (5)} \\ & E_s(h_0) \le \beta\tilde{E}_{s0}, \quad E_p(h_0) \le \beta\tilde{E}_{p0}, \\ & E_s(h_1) \le \beta\tilde{E}_{s1}, \quad E_p(h_1) \le \beta\tilde{E}_{p1}. \end{array} \tag{8}$$

where β is a control parameter indicating how much stopband and passband energies can be relaxed for the coding gain. Here, we set $\beta = 1.1$. The reasons for this small β are as follows: a) The strong assumptions in deriving (6) may not be valid for some images. b) The optimization of coding gain in the first stage of subband decomposition is not very accurate.

IMPLEMENTATION AND CODING QUALITY

To solve the two-stage optimization problem, we utilize an SQP (sequential quadratic programming) package called FSQP (Feasible SQP) [8]. We set the maximum iterations of FSQP to be 1,000, and the convergence precision for equality constraints to be 10^{-15}. Since the objective and constraints are nonlinear, (7) and (8) may have lots of local minima. To get better designs, we randomly generate 100 starting points within the search space, $-1 \leq h_0(n), h_1(n) \leq +1$. For each starting point, we do local search using FSQP, and pick the best solution with the smallest objective value.

Three filter banks of different lengths (N_0, N_1) were designed: a) (9,7), b) (13,7), and c) (13,11). The cut-off frequencies ω_s and ω_p control how close the designed filters are to the ideal filters. For the (9,7) filter bank, we set $\omega_s = 0.7\pi$ and $\omega_p = 0.3\pi$. For the other two filter banks, we set $\omega_s = 0.6\pi$ and $\omega_p = 0.4\pi$.

For each filter bank designed, we compare its performance with that of Antonini's (9,7) filter bank that was reported to perform the best [7]. We implemented both schemes using a wavelet image coding package [3] with the same quantizers and entropy coders. We tested two images of different styles: a smooth image (Lena), and a more detailed one with some textures (Barbara). For each image, we tried the following compression ratios: 4:1, 8:1, 16:1 and 32:1, and calculated their peak signal-to-noise ratios (PSNRs).

Figure 2 compares the coding qualities in terms of PSNRs and compression ratios, where the (9,7), (13,7) and (13,11) filters were designed by our method. For Lena, our filter banks have similar performance as Antonini's filter bank, with a maximum improvement of 0.2 dB at low compression ratio 4:1. For Barbara, our filter banks perform much better: (9,7) improves by up to 0.65 dB, (13,7) by up to 0.75 dB, and (13,11) by up to 0.75 dB.

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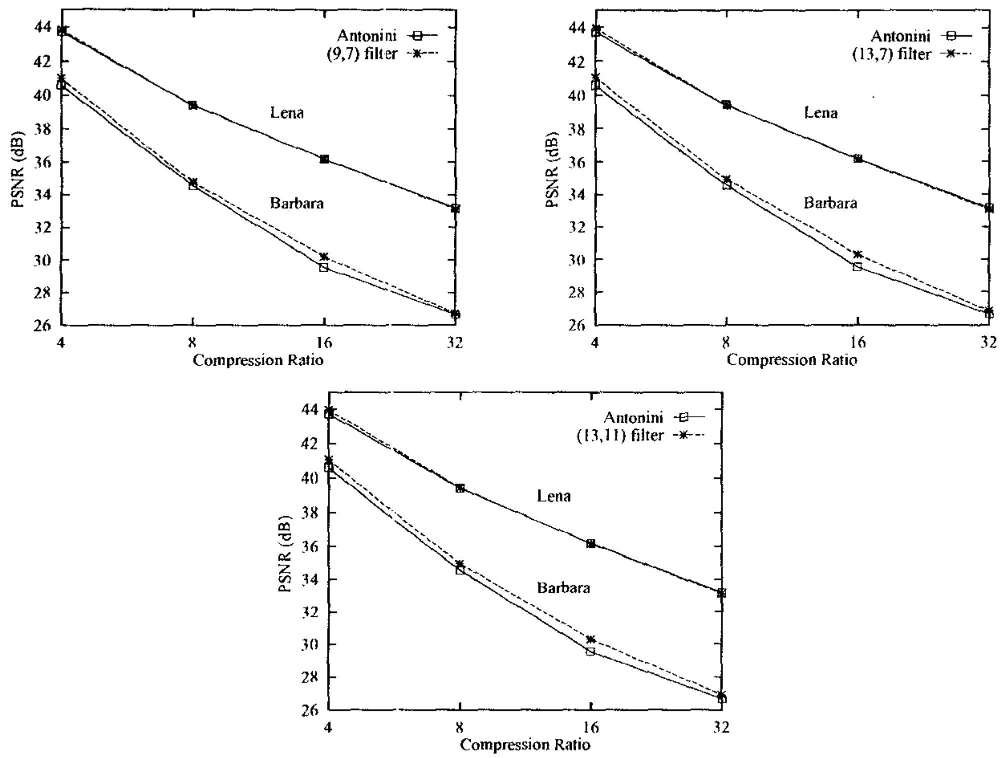

Figure 2: Comparison of our designed filter banks and Antonini's filter bank.

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