AUTOMATED PLANNING AND SCHEDULING USING CALCULUS OF VARIATIONS IN DISCRETE SPACE

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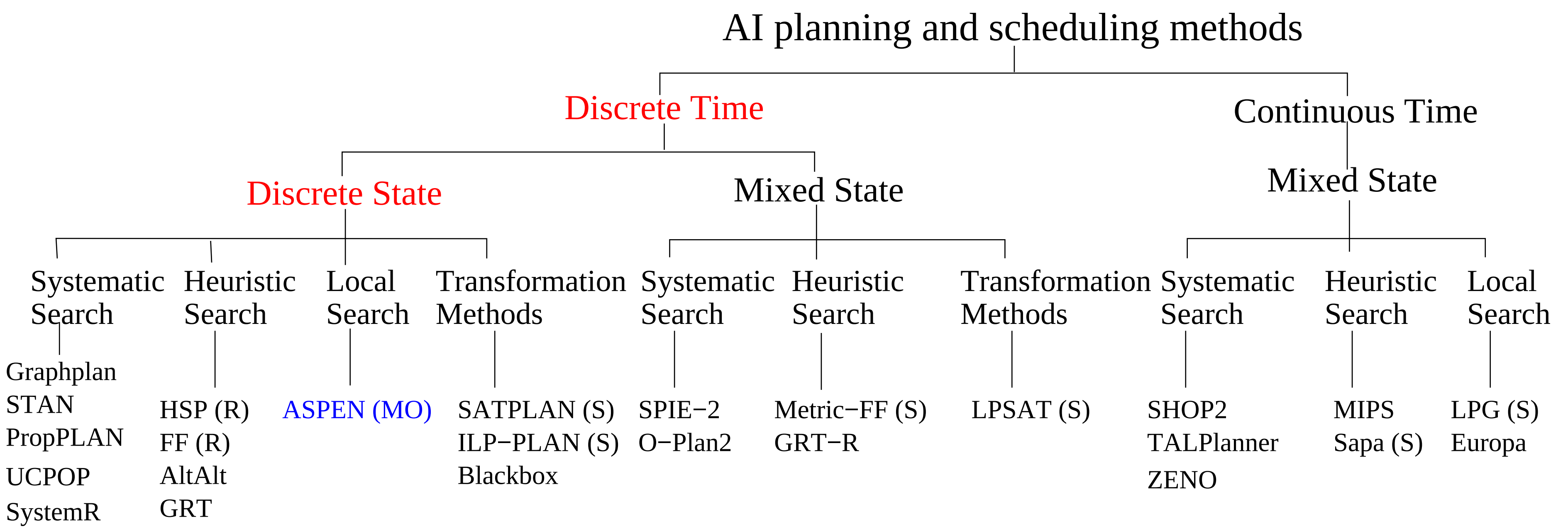
Outline

- Introduction
  - Research problem addressed
  - Limitations of dynamic programming and penalty methods
- Theory of Lagrange multipliers for discrete constrained optimization
  - Necessary and sufficient saddle-point condition
  - Iterative implementation
- Partitioning of variable space
  - Distributed necessary saddle-point condition
  - Distributed Iterative implementation
- Experimental results on ASPEN
- Conclusions

INTRODUCTION

A Classification of Existing Approaches in Planning

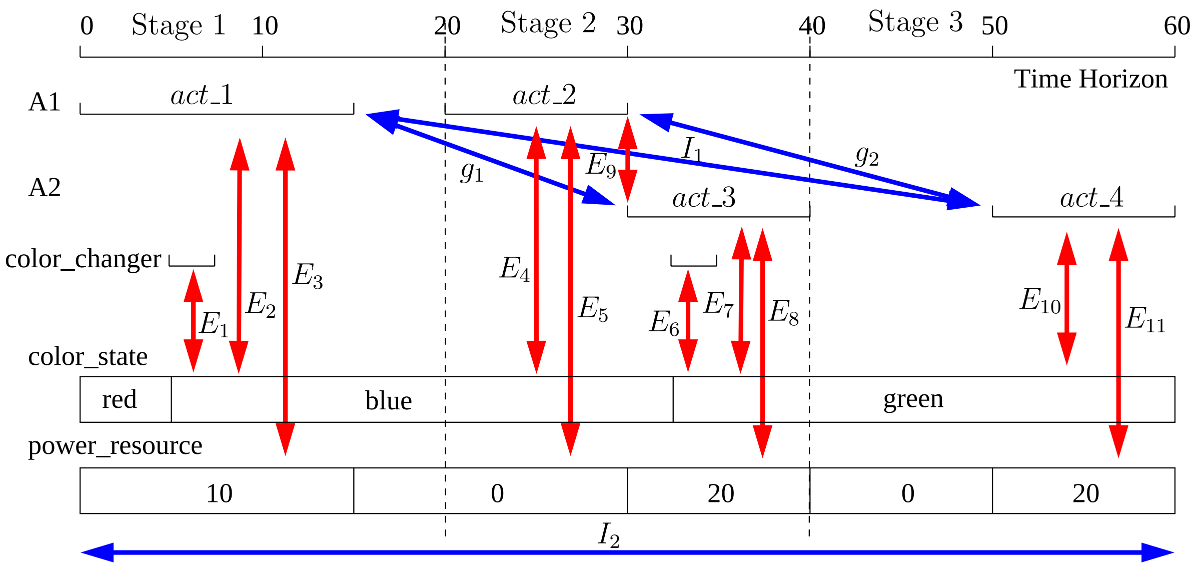


ASPEN

- Discrete time horizon and discrete space
- Discrete temporal and metric constraints
- Multiple preferences combined in a weighted sum
- Greedy local optimization of objective and repair-based constraint satisfaction

Toy Example solved by ASPEN

```
model toy {HORIZON_START = 1998-1/00:00:00; horizon_duration = 60s; time_scale = second;};
parameter string color {domain = ("red", "blue", "green");};
State_variable color_sv {states = ("red", "blue", "green"); default_state = "red";};
Resource power {type = non_depletable; capacity = 25; min_value = 0;};
Activity color_changer {color c; duration = 1; reservations = color_sv change_to c;};
Activity A1 {duration = [10,20]; constraints = ends_before start of A2 by [0,30];
       reservations = power use 10, color_sv must_be "green";};
Activity A2 {duration = 10; reservations = power use 20, color_sv must_be "blue";};
// initial schedule
A1 act_1 {start_time = 0; duration = 15;};   act_2 { start_time = 20; duration = 10;};
A2 act_3 {start_time = 30; duration = 10;};  act_4 { start_time = 50; duration = 10;};
```

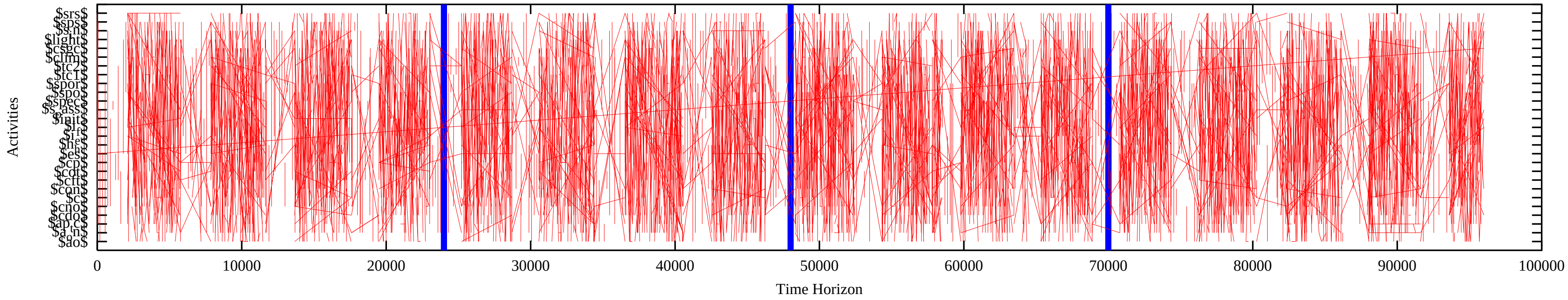


Local constraints ($E_1, \cdots, E_{11}$):
- color_state constraints for *act*_1, act_2, *act*_3,
- power_resource constraints for *act*_1 thru *act*_4;
- color_state transition constraints relating color_changer and color_state
- *act*_2 ends_before start of *act*_3 by [0,30].

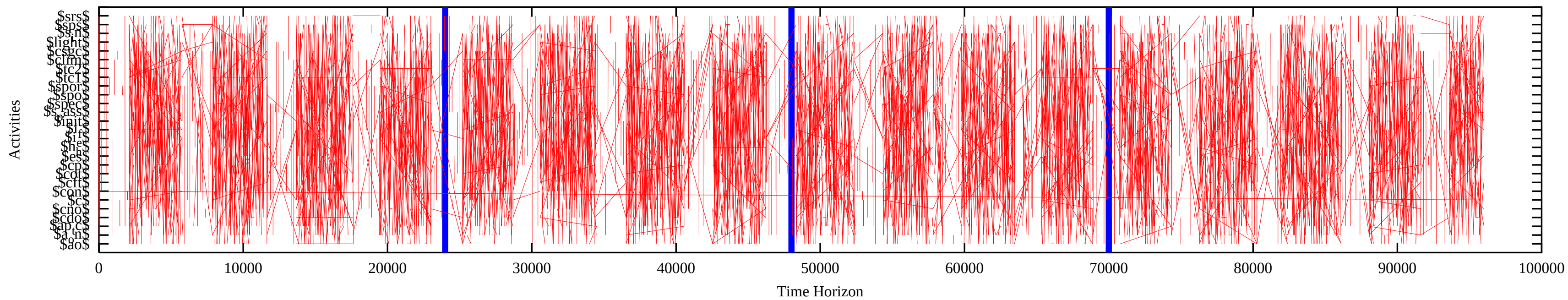
General constraints:
- *act*_1 ends_before start of *act*_3 by [0,30] (g_1);
- *act*_2 ends_before start of *act*_4 by [0,30] (g_2).
- *act*_1 ends_before start of *act*_4 by [0,30] (I_1);
- power_resource always less than capacity of power_resource (I_2).

Solving CX1-PREF with 16 Orbits (with 3,687 constraints) by ASPEN

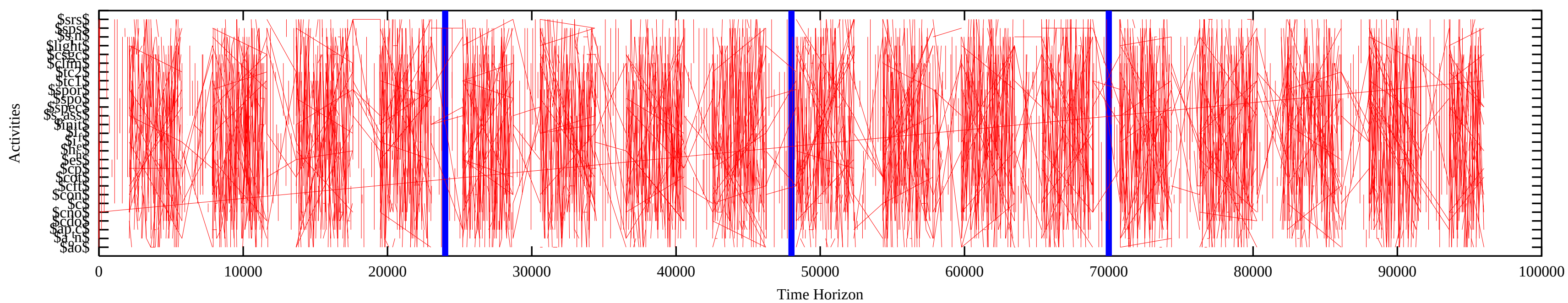
Initially



1,000 Iter.



1,0000 Iter.

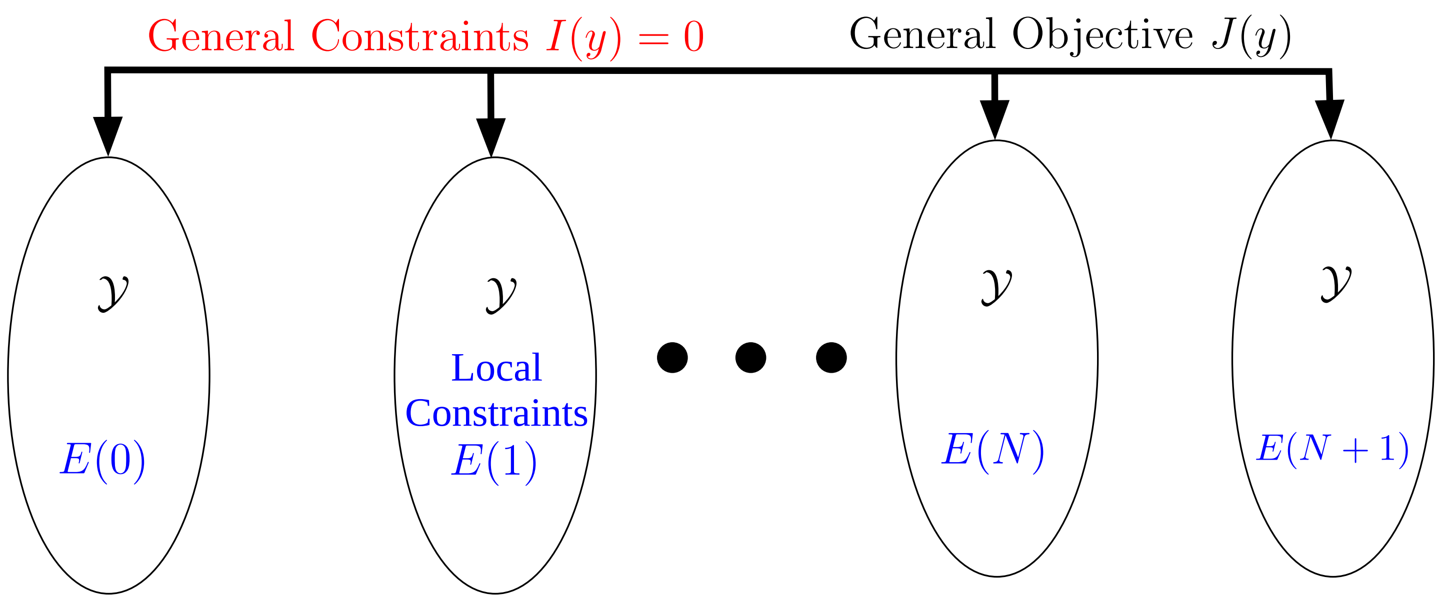


Research Problem Addressed

Partition (discrete-space and discrete-time) temporal planning problems and develop methods for resolving general constraints across partitions

- Partitioned problems have lower time and space complexity
- Overall problem can be solved better and more efficiently

Mathematical Formulation

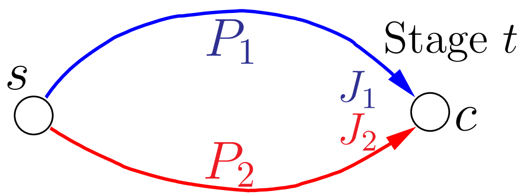


$$\begin{aligned} \min_{y} \quad & J(y) \\ \text{subject to} \quad & E(j, y(j)) = 0, \quad j = 0, 1, \cdots, N+1 \\ & I(y) = 0 \end{aligned}$$

where $y(j)$ is defined in *discrete* space $\mathcal{Y}$ of stage j,
E, I and J are *not* necessarily continuous and differentiable

Dynamic Programming Cannot Be Applied

- Path dominance on multi-stage search with local constraints
  - Principle of Optimality applied on feasible state c



If c lies on the optimal path between s and d and $J_2 \leq J_1 \quad \Longrightarrow \quad P_2 \rightarrow P_1$

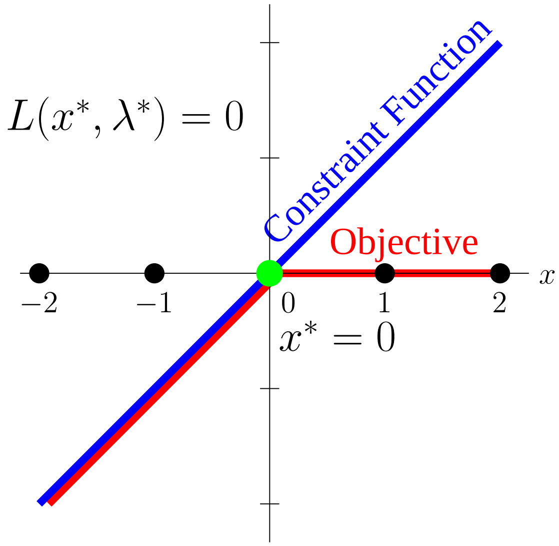
  - Polynomial worst-case complexity: $O\left(N|\mathcal{Y}|^2\right)$
- Path dominance is not applicable when there are general constraints
  - A dominating path may become infeasible due to general constraints
  - Exponential search space: $O\left(|\mathcal{Y}|^{N+2}\right)$

Penalty-Based Methods Do Not Always Work

Penalty-based methods

- By choosing suitable penalties in a penalty function, a local minimum of the penalty function corresponds to a feasible local minimum of the objective

Counter-example



$$\min_{x \in \{-2,-1,0,1,2\}} f(x) = \begin{cases} 0 & x \geq 0 \\ x & x < 0 \end{cases}$$

subject to $x = 0$

Penalty formulation

- $L(x, \lambda) = f(x) + \lambda\, x$
- Hypothesize $L(x, \lambda^*) \geq L(x^*, \lambda^*) = 0$

No λ^* exists when solving

$$\begin{cases} L(-1, \lambda^*) = -1 - \lambda^* \geq L(0, \lambda^*) = 0 \\ L(1, \lambda^*) = 0 + \lambda^* \geq L(0, \lambda^*) = 0 \end{cases}$$

$$\Longrightarrow \begin{cases} \lambda^* \leq -1 \\ \lambda^* \geq 0 \end{cases}$$

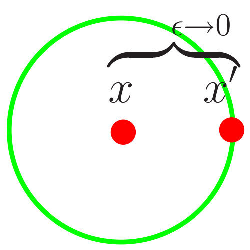
THEORY OF LAGRANGE MULTIPLIERS FOR DISCRETE CONSTRAINED OPTIMIZATION

Neighborhood $\mathcal{N}(x)$ of Point x

Continuous Space: $\mathcal{N}_{cn}(x)$

x is a vector of continuous variables

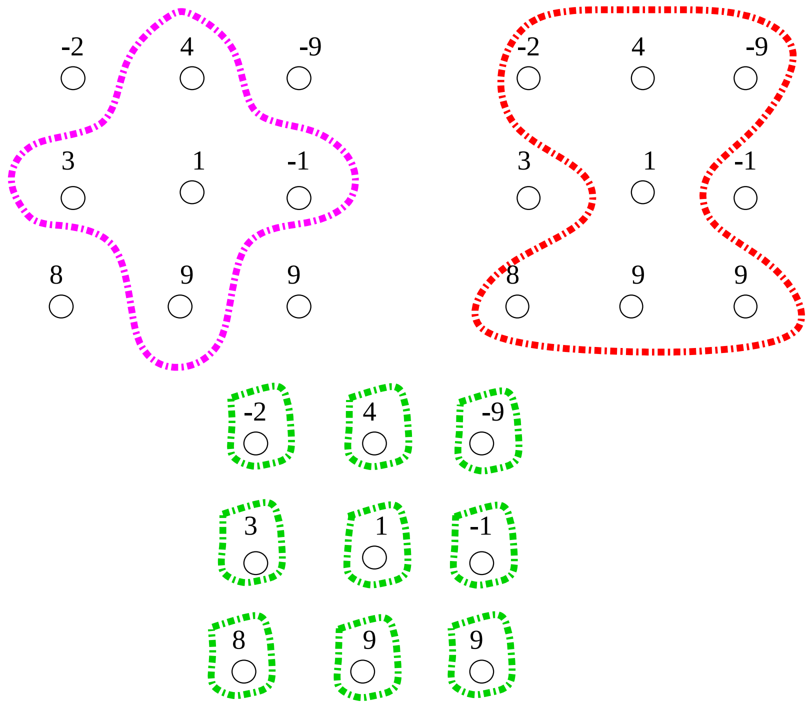
Neighborhood defined by open sphere



Discrete Space: $\mathcal{N}_{dn}(x)$

x is a vector of discrete variables

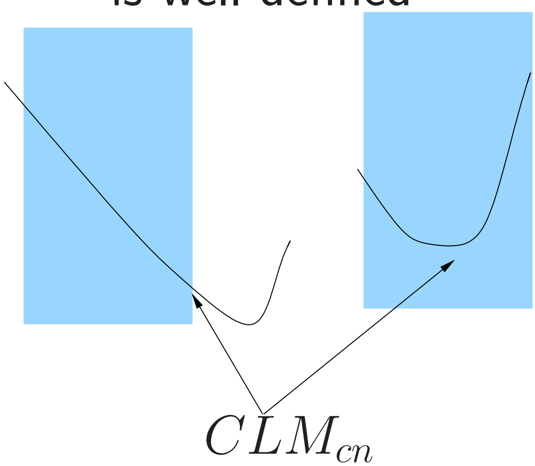
User defined neighborhood



Constrained Local Minimum (CLM)

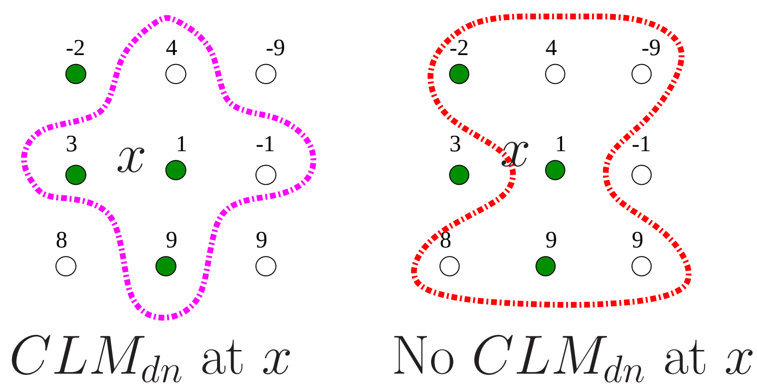
Continuous Space: CLM_{cn}

- Feasible local minimum when compared to feasible points inside an open sphere
- Whether point x is a CLM_{cn} is well defined



Discrete Space: CLM_{dn}

- Feasible local minimum with respect to neighboring feasible points
- Whether point x is a CLM_{dn} depends on $\mathcal{N}_{dn}(x)$



Lagrangian Formulation of Discrete Optimization Problem

- Let H be a transformation function where $H(x) \geq 0$ and $H(x) = 0$ iff $x = 0$

$$L_{dn}(y, \gamma, \mu) = J(y) + \sum_{t=0}^{N+1} \gamma^T(t) H\left(E(t, y(t))\right) + \mu^T H\left(I(y)\right)$$

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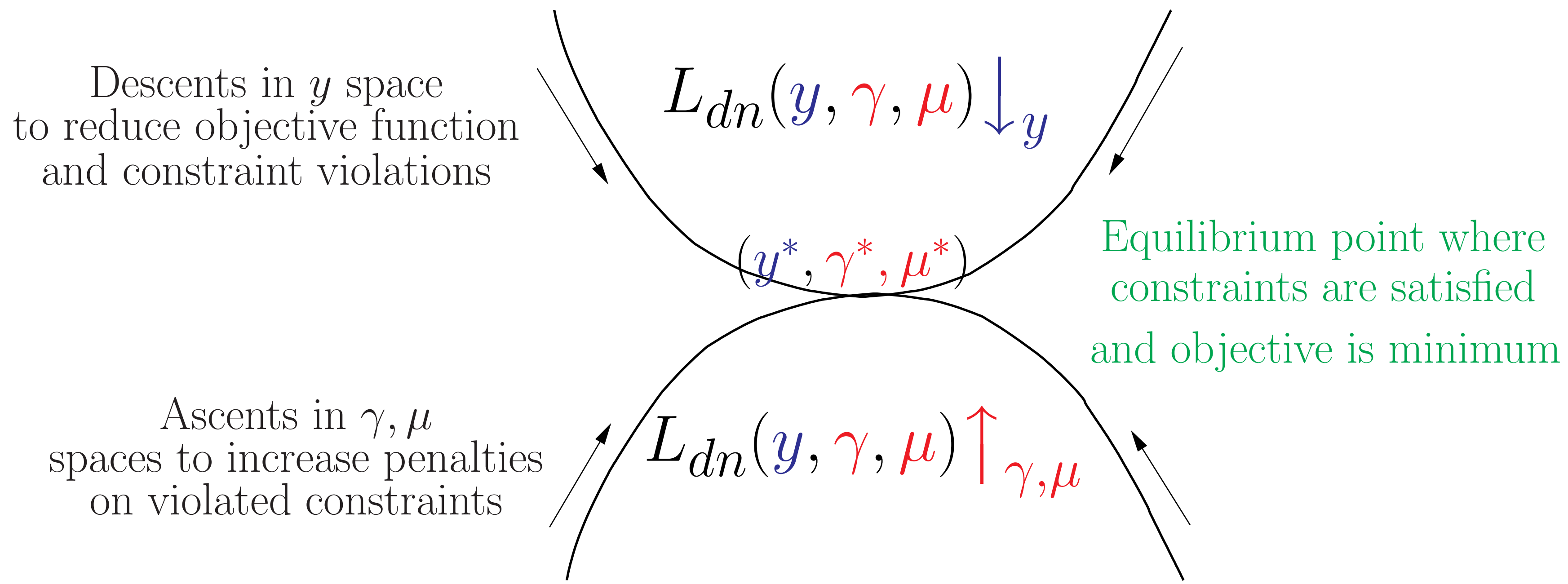
- Necessary and sufficient saddle-point condition
  - y^* is a CLM_{dn} iff (y^*, γ^*, μ^*) is a discrete-neighborhood saddle point (SP_{dn})

$$L_{dn}(y^*, \gamma, \mu) \leq L_{dn}(y^*, \gamma^*, \mu^*) \leq L_{dn}(y, \gamma^*, \mu^*)$$

- (y^*, γ^*, μ^*) is at
  - Local minimum of L_{dn} with respect to y
  - Local maximum of L_{dn} with respect to γ and μ
- Condition is true for $\gamma^{**} \geq \gamma^*$ and $\mu^{**} \geq \mu^*$

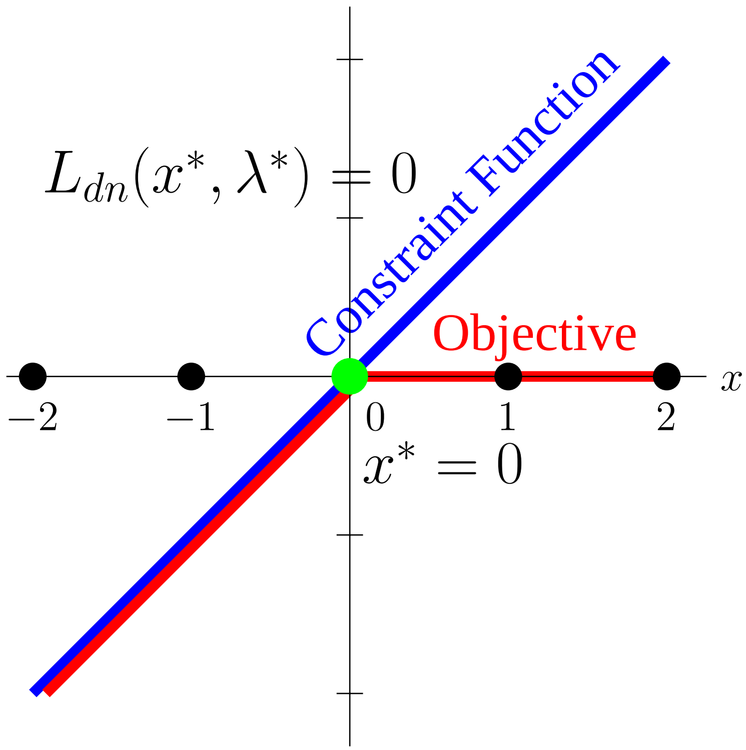
Intuitive Meaning Behind Saddle Points



Although γ^* and μ^* always exists,

- Their search in discrete space may be very time consuming
- The search of $\gamma^{**} \geq \gamma^*$ and $\mu^{**} \geq \mu^*$ is much easier

Continuing from the Previous Example



$$\min_{x \in \{-2,-1,0,1,2\}} f(x) = \begin{cases} 0 & x \geq 0 \\ x & x < 0 \end{cases}$$

subject to $x = 0$

Lagrangian formulation

- $L_{dn}(x, \lambda) = f(x) + \lambda\,|x|$
- Find λ^* such that $L_{dn}(x, \lambda^*) \geq L_{dn}(x^*, \lambda^*)$

Solving

$$\begin{cases} L_{dn}(-1, \lambda^*) = -1 + \lambda^* \geq L_{dn}(0, \lambda^*) = 0 \\ L_{dn}(1, \lambda^*) = 0 + \lambda^* \geq L_{dn}(0, \lambda^*) = 0 \end{cases}$$

leads to $\lambda^* \geq 1$

Pick $\lambda^* = 1$
Saddle-point condition applies for $\lambda^{**} \geq \lambda^*$

Iterative Implementation

Algorithm needs to look for $\gamma^{**} \geq \gamma^*$ and $\mu^{**} \geq \mu^*$

$L_{dn}(y, \gamma, \mu) \uparrow_{\gamma,\mu}$ to find γ^{**}, μ^{**}

Outer Loop

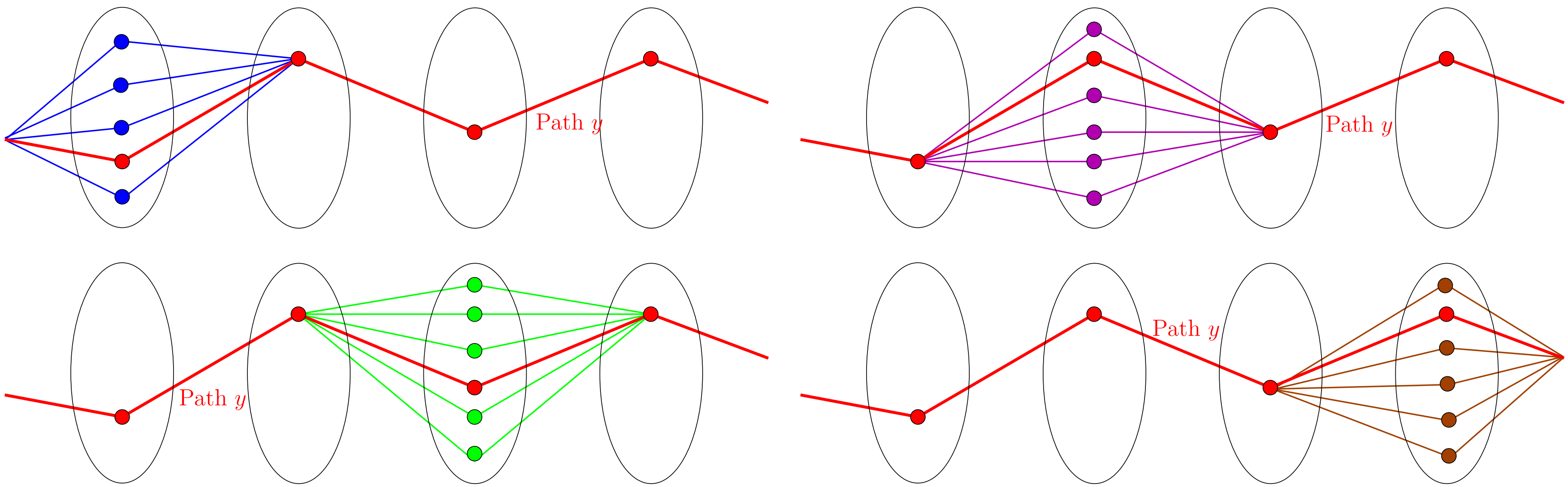
$L_{dn}(y, \gamma, \mu) \downarrow_{y}$ to find y^*

Inner Loop

PARTITIONING OF VARIABLE SPACE

Neighborhood in Partitioned Variable Space

$\mathcal{N}_b(y)$ (discrete neighborhood of path $y = (y(0), \ldots, y(N+1))^T$) is the union of discrete neighborhoods in each stage, while keeping the path fixed in other stages



Path y is a constrained local minimum in discrete space (CLM_{dn}) iff

- y is feasible
- No feasible path in $\mathcal{N}_b(y)$ has better objective value than $J(y)$

Decomposition of Lagrangian Function into Stages

Decompose Lagrangian function

$$L_{dn}(y, \gamma, \mu) = J(y) + \sum_{t=0}^{N+1} \gamma^T(t) \cdot \big|E(t, y(t))\big| + \mu^T \cdot \big|I(y)\big|$$

into distributed Lagrangian function for stage t, $t = 0, \cdots, N+1$,

$$\Gamma_{dn}(t, y, \gamma(t), \mu) \;=\; J(y) + \gamma(t) \cdot \big|E(t, y(t))\big| + \mu \cdot \big|I(y)\big|$$

Distributed Necessary Conditions for CLM_{dn}

- Path y is a CLM_{dn} when it satisfies
  - Distributed necessary discrete-neighborhood saddle-point conditions

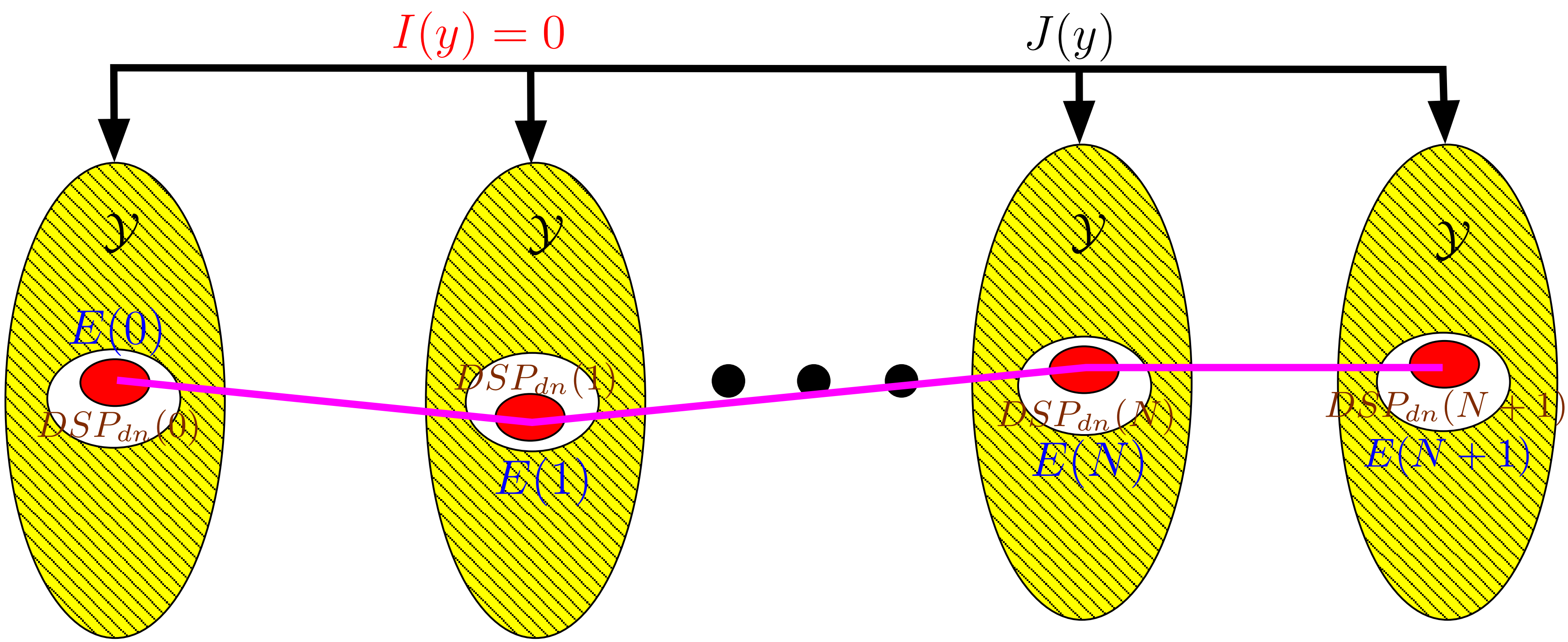
    $(DSP_{dn}(t))$ for all $t = 0, 1, \cdots, N+1$ and fixed μ^*

$$\Gamma_{dn}(t, y^*, \gamma(t)', \mu^*) \leq \Gamma_{dn}(t, y^*, \gamma(t)^*, \mu^*) \leq \Gamma_{dn}(t, y', \gamma(t)^*, \mu^*)$$

$$\text{for all } y' = (y(0), \ldots, y(t-1), y(t)', y(t+1), \ldots, y(N+1)) \in \mathcal{N}_b^{(t)}(y^*)$$

  - Condition is also true for all $\gamma(t)^{**} \geq \gamma(t)^*$

Reduced Search Space for Finding Feasible/Optimal Paths



Significant reduction in complexity

Iterative Implementation

$L_{dn}(y, \gamma, \mu)\uparrow_{\mu}$ to find μ^{**}

$\Gamma_{dn}(0, y, \gamma, \mu)\uparrow_{\gamma(0)}$ to find $\gamma^{**}(0)$

$\Gamma_{dn}(0, y, \gamma, \mu)\downarrow_{y(0)}$

• • •

$\Gamma_{dn}(N+1, y, \gamma, \mu)\uparrow_{\gamma(N+1)}$ to find $\gamma^{**}(N+1)$

$\Gamma_{dn}(N+1, y, \gamma, \mu)\downarrow_{y(N+1)}$

Observation:

- Based on the neighborhood of partitioned variable space, the combined local minimum of Γ_{dn} in all subspaces is the local minimum of L_{dn}

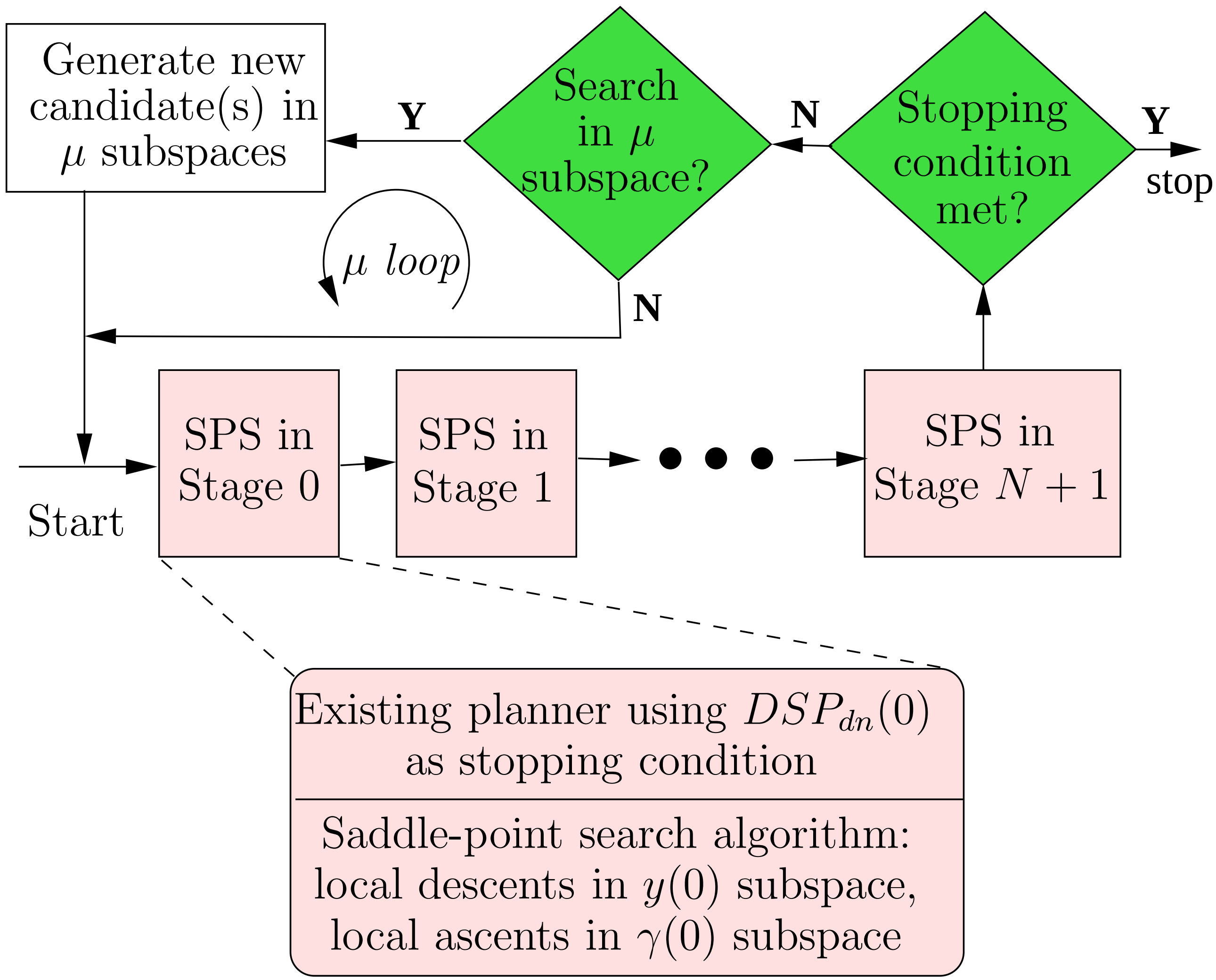
Equivalent Search:

$L_{dn}(y, \gamma, \mu)\uparrow_{\mu}$ to find μ^{**}

$L_{dn}(y, \gamma, \mu)\uparrow_{\gamma}$ to find γ^{**}

$L_{dn}(y, \gamma, \mu)\downarrow_{y}$ to find y^{*}

Heuristic Search Procedure for Finding Local Optimal Paths



EXPERIMENTAL RESULTS ON ASPEN

Approach

1. Dynamically partition problem into $N = 100$ stages with a balanced number of conflicts in each
2. In each stage, perform a certain number of descents and ascents
   - Choose probabilistically from repair actions and optimization actions, and select a random feasible choice at each choice point to create an action
   - Apply action using ASPEN
   - Evaluate augmented distributed Lagrangian function of stage t:

$$\Gamma_{dn}(t) = -w_s \cdot \text{Score} + \sum_{i \in E(t,y(t))} \gamma_i \cdot c_i + \sum_{i \in E(t,y(t))} \frac{c_i^2}{2} + \sum_{j \in I(y)} \mu_j \cdot d_j$$

Score $\in [0, 1]$: preference score of schedule
$w_s = 100$: weight of Score
$c_i = 1$: non-negative value on the degree of violation of local conflict i
$d_j = 1$: non-negative value on the degree of violation of global conflict j

- Accept schedule according to Metropolis probability controlled by a geometrically decreasing T (initial $T = 1000$; cooling rate $= 0.8$)
- For each descent, perform an ascent in γ_i space on violated local conflicts

$$\gamma_i \longleftarrow \gamma_i + \alpha_i \cdot c_i \qquad \text{where } \alpha_i = 0.1$$

3. After iterating over all stages, perform an ascent in μ_j space on violated global conflicts

$$\mu_j \longleftarrow \mu_j + \alpha_j \cdot d_j \qquad \text{where } \alpha_j = 0.1$$

4. If maximum number of iterations is not exceeded, go to (1)

Without UNDO in ASPEN

- Apply selected action to current schedule in a child process
- Repeat the same action in the parent process if action is accepted;
- Discard the result of the child process
- The number of forks is OS dependent and set to 24,000 in our experiments

Benchmark CX1-PREF

- Citizen Explorer-I satellite design and operation planning benchmark
  - Multiple competing preferences to be optimized
  - Problem generator to generate different problem instances

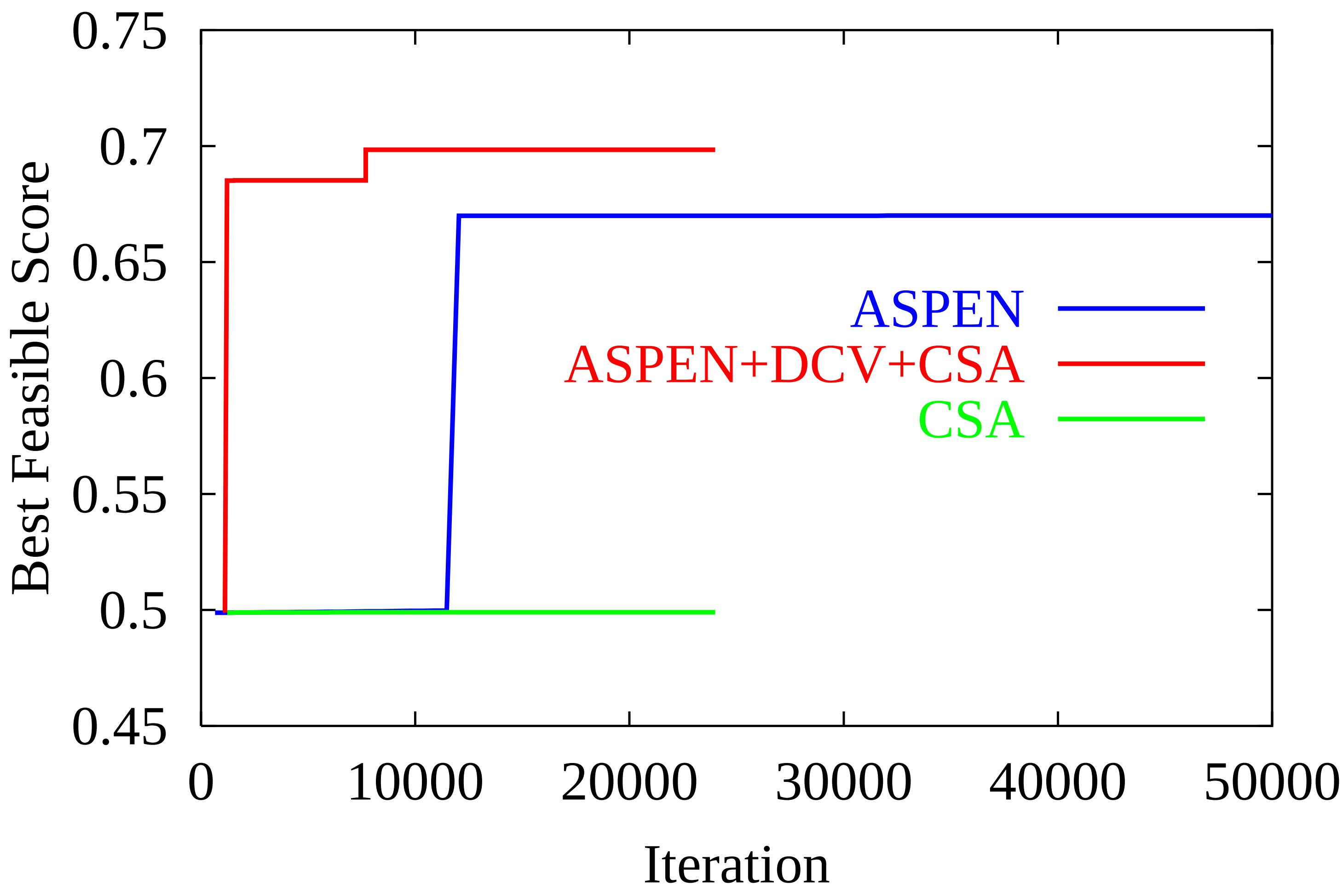
  $perl \quad probgen.pl \quad < random\ seed > \quad < number\ of\ orbits >$

- In default ASPEN search, repeat the following steps

  a) Find feasible schedule using *repair*

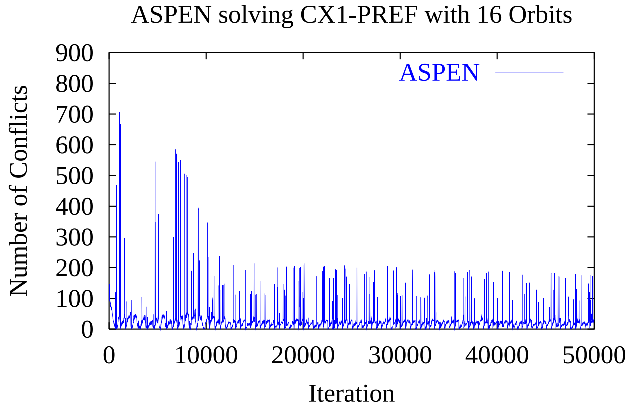
  b) Optimize score using *optimize* (default 200 iterations)

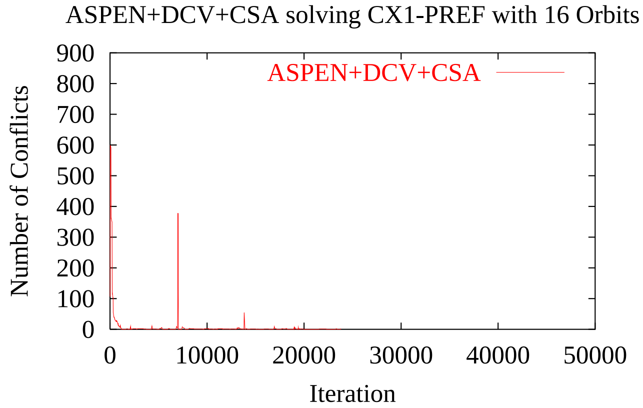
Best Feasible Solution on CX1-PREF with 16 Orbits



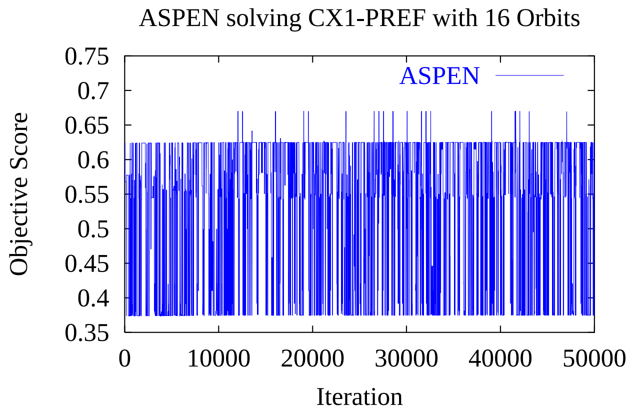
Search Progress on CX1-PREF with 16 Orbits

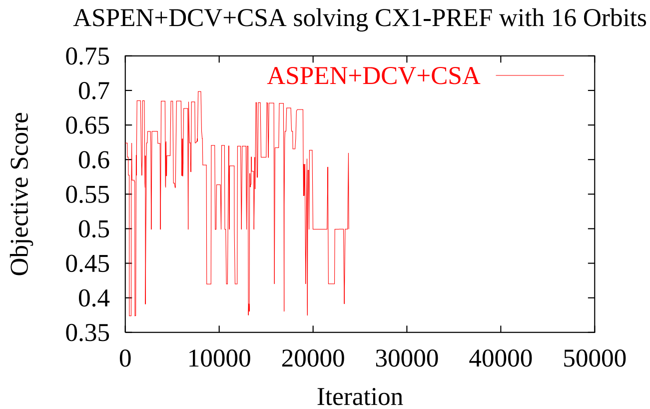
- Conflicts versus Iteration:



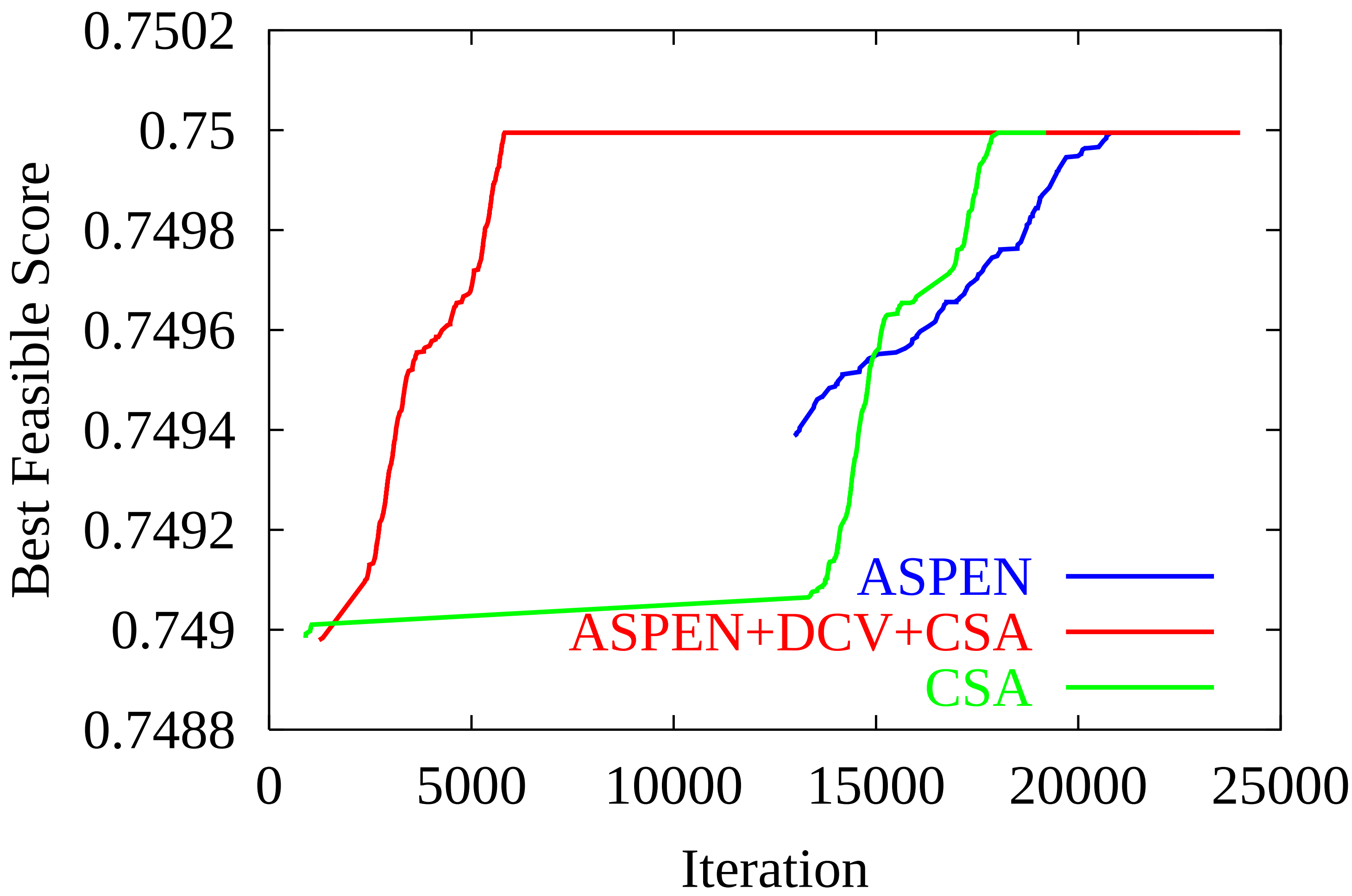


- Score versus Iteration:

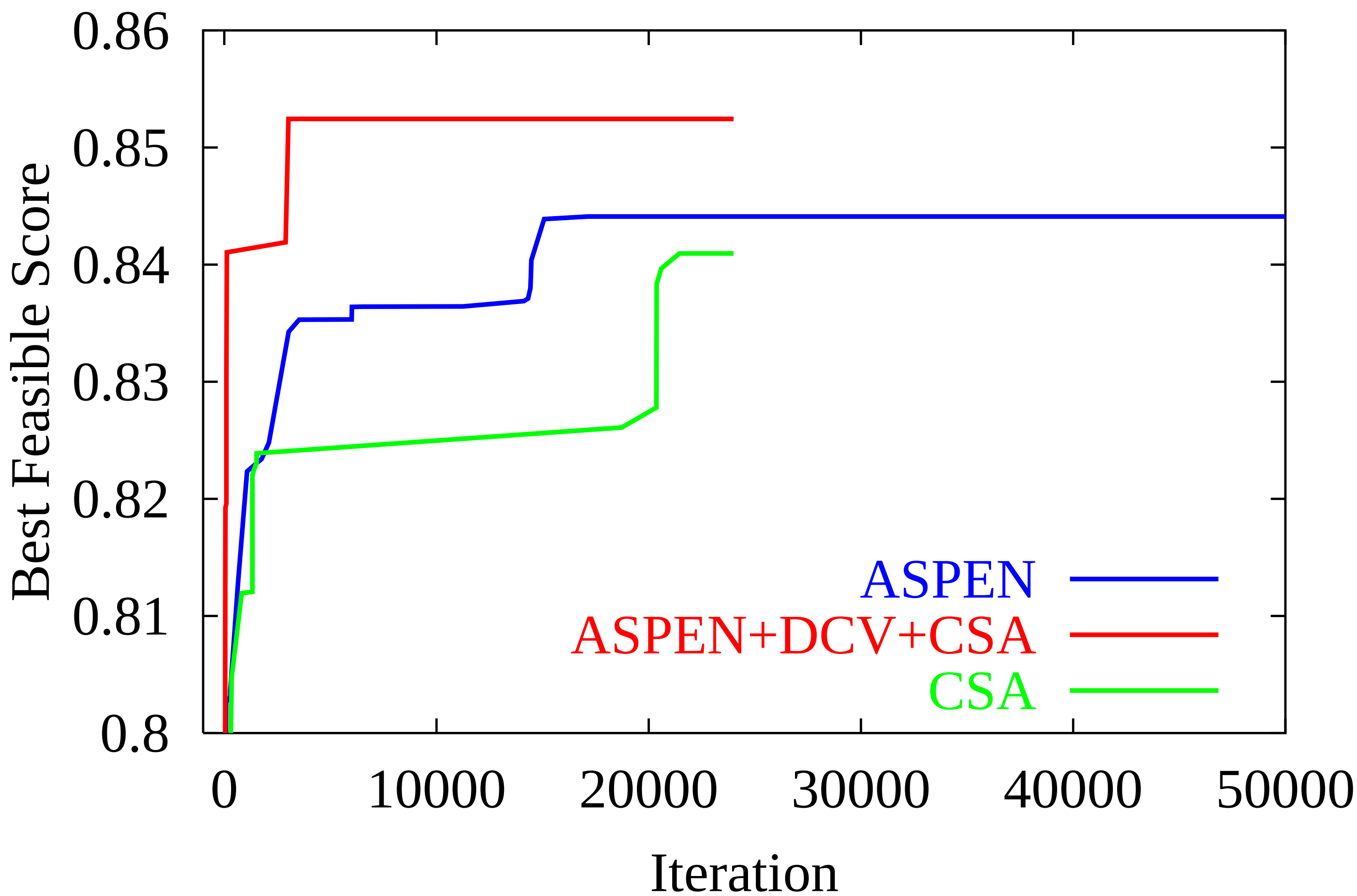




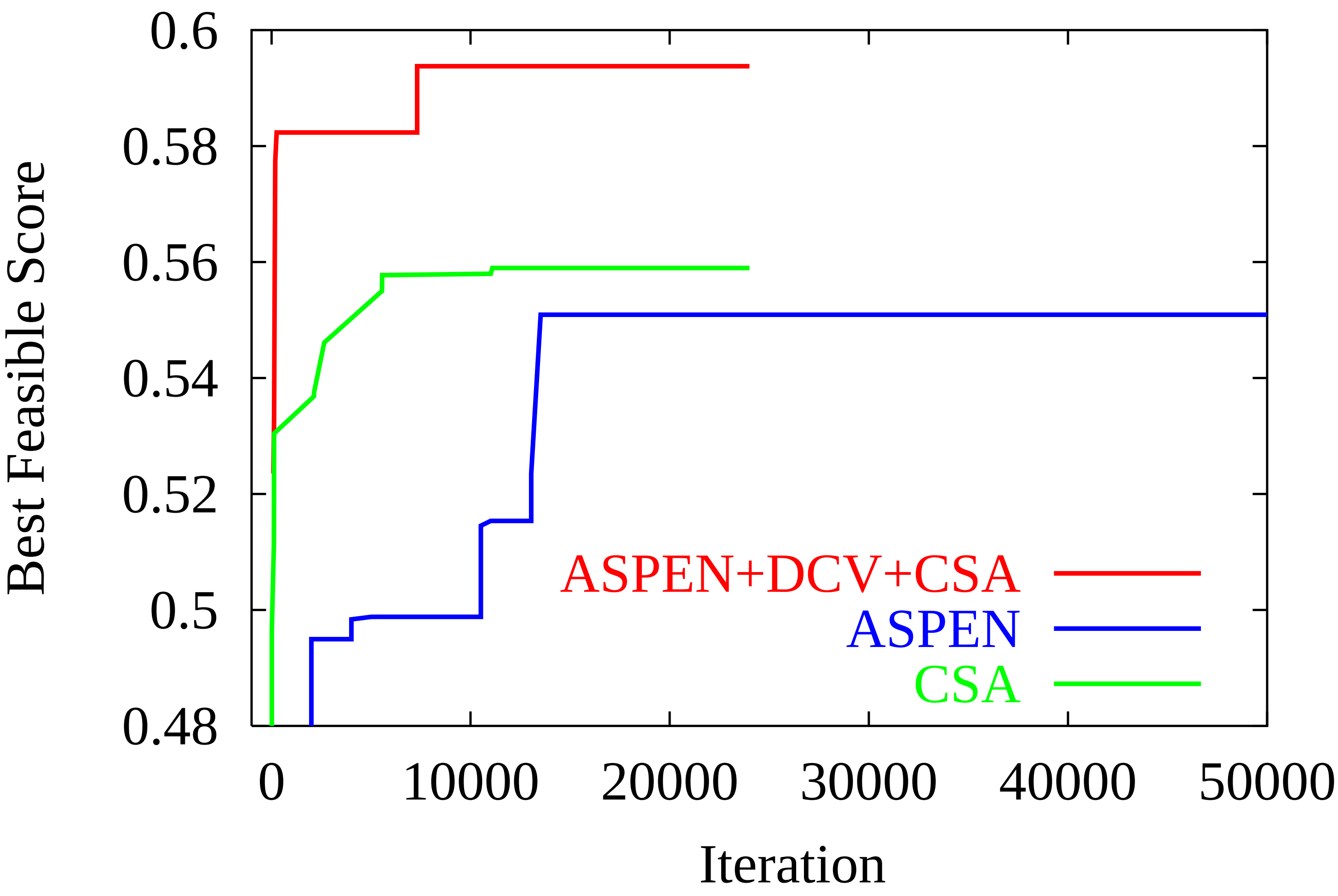
Best Feasible Solution on CX1-PREF with 8 Orbits



Best Feasible Solution on OPTIMIZE Benchmark



Best Feasible Solution on PREF Benchmark



Conclusions

- Partitioning of discrete constrained optimization in temporal planning
  - Systematic method to resolve general constraints across partitions
  - Significant reduction in search space by reducing the base of the exponential complexity
- Few parameters to tune in algorithm
- Extensions to temporal planning in continuous and mixed domains