PARTITIONING OF TEMPORAL PLANNING PROBLEMS IN MIXED SPACE USING THE THEORY OF EXTENDED SADDLE POINTS

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Outline

• Research problem addressed
  – Limitation of existing planning methods

• Theory of Lagrange multipliers for mixed constrained optimization
  – Necessary and sufficient extended saddle-point condition
  – Iterative implementations

• Partitioning of variable space
  – Distributed extended saddle-point condition
  – Distributed Iterative implementations

• Experimental results on MIPS and PDDL2.1

• Conclusions
A Classification of Existing Approaches in Planning

AI planning and scheduling methods

Discrete Time

- Discrete State
  - Systematic Search
  - Heuristic Search
  - Local Search
  - Transformation Methods
- Mixed State
  - Systematic Search
  - Heuristic Search
  - Transformation Methods

Continuous Time

- Mixed State
  - Systematic Search
  - Heuristic Search
  - Local Search

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Research Problem Addressed

Partition (mixed-space and continuous-time) temporal planning problems and develop methods for resolving global constraints across partitions

- Partitioned problems have lower time and space complexity
- Overall problem can be solved better and more efficiently
Mathematical Formulation

\[ \min_{z} J(z) \]
subject to \( E(z(j)) = 0, \quad j = 0, 1, \ldots, N + 1 \)
\( I(z) = 0 \)

where \( z(j) \) is defined in mixed space \( \mathcal{Z} \) of stage \( j \),
\( E, I \) and \( J_i \) are not necessarily continuous and differentiable

Dynamic Programming Cannot Be Applied

- Path dominance on multi-stage search with local constraints
  - Principle of Optimality applied on feasible state \( c \)
    \[ \begin{array}{c}
    s \\
    \quad \scriptsize \text{Stage } t
    \end{array} \begin{array}{c}
    P_1 \\
    \quad \scriptsize \text{If } c \text{ lies on the optimal path between } s \text{ and } d \text{ and } \\
    J_2 \leq J_1 \quad \Rightarrow \\
    P_2 \rightarrow P_1
    \end{array} \\
    \begin{array}{c}
    c \\
    \quad \scriptsize \text{P} \_2
    \end{array} \]
  - Polynomial worst-case complexity: \( O \left( N|\mathcal{Z}|^2 \right) \)

- Path dominance is not applicable when there are global constraints
  - A dominating path early on may become infeasible due to global constraints that got violated later
  - Exponential search space: \( O \left( |\mathcal{Z}|^{N+2} \right) \)
Penalty-Based Methods Do Not Always Work

Penalty-based methods
- By choosing suitable penalties in a penalty function, a local minimum of the penalty function corresponds to a feasible local minimum of the objective.

Counter-example

Penalty formulation
- \( L(x, \lambda) = f(x) + \lambda x \)
- Hypothesize \( L(x, \lambda^*) \geq L(x^*, \lambda^*) = 0 \)

No \( \lambda^* \) exists when solving

\[
\begin{align*}
0 &= L(0, \lambda^*) \leq L(-1, \lambda^*) \\
0 &= L(0, \lambda^*) \leq L(1, \lambda^*)
\end{align*}
\]

\( \Rightarrow \)

\[
\begin{align*}
\lambda^* \leq -1 \\
\lambda^* \geq 0
\end{align*}
\]

Mathematical Programming Methods

Continuous Methods: unique \( \lambda \) cannot be found in distributed methods
- Necessary KKT condition
- Sufficient saddle point condition

MINLP methods: require the function of subproblems to be convex or factorable
- Generalized Benders Decomposition
- Outer Approximation
- Branch-and-Reduce Methods
THEORY OF EXTENDED SADDLE POINTS FOR MIXED CONSTRAINED OPTIMIZATION

**Mixed Neighborhood \( N_m(z) \) of Point \( z \)**

\[
N_m(z) = N_m(x, y) = \{ (x', y) \mid x' \in N_c(x) \} \cup \{ (x, y') \mid y' \in N_d(y) \}
\]

- **Continuous Subspace:** \( N_c(x) \)
- **Discrete Subspace:** \( N_d(y) \)

- \( x \) is a vector of continuous variables
- \( y \) is a vector of discrete variables
- Neighborhood defined by open sphere
- User defined neighborhood

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Constrained Local Minimum (CLM)

- Feasible $z$ is $CLM_m$ in mixed space if $J(z) \leq J(z') \forall$ feasible $z' \in N_m(z)$

Continuous Subspace: $CLM_c$

- Feasible local minimum when compared to feasible points inside an open sphere
- Whether point $x$ is a $CLM_c$ is well defined

Discrete Subspace: $CLM_d$

- Feasible local minimum with respect to neighboring feasible points
- Whether point $y$ is a $CLM_d$ depends on $N_d(y)$

Lagrangian Formulation of Mixed Optimization Problem

- Transformed Lagrangian function with extended Lagrange multipliers $\gamma$ and $\mu$

$$L_m(z, \gamma, \mu) = J(z) + \sum_{t=0}^{N+1} \gamma^T(t) \cdot \left| E(z(t)) \right| + \mu^T \cdot \left| I(z) \right|$$

- Necessary and sufficient Extended Saddle-Point Condition (ESPC)
  - $z^*$ is a $CLM_m$ iff $(z^*, \gamma^*, \mu^*)$ is a mixed-neighborhood saddle point $(SP_m)$
  $$L_m(z^*, \gamma, \mu) \leq L_m(z^*, \gamma^*, \mu^*) \leq L_m(z, \gamma^*, \mu^*)$$

- $(z^*, \gamma^*, \mu^*)$ is at
  - Local minimum of $L_m$ with respect to $z$
  - Local maximum of $L_m$ with respect to $\gamma$ and $\mu$

- Condition is true for $\gamma^{**} > \gamma^*$ and $\mu^{**} > \mu^*$
### Intuitive Meaning Behind Saddle Points

Descents in $z$ space to reduce objective function and constraint violations

Ascents in $\gamma, \mu$ spaces to increase penalties on violated constraints

Equilibrium point where constraints are satisfied and objective is minimum

Although $\gamma^*$ and $\mu^*$ always exists,

- Their search in mixed space may be very time consuming
- The search of $\gamma^{**} > \gamma^*$ and $\mu^{**} > \mu^*$ is much easier

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### Continuing from the Previous Example

**Transformed Lagrangian function**

- $L_d(x, \lambda) = f(x) + \lambda |x|$

- Find $\lambda^*$ such that $L_d(x, \lambda^*) \geq L_d(x^*, \lambda^*)$

Solving

$$
\begin{align*}
0 &= L_d(0, \lambda^*) \leq L_d(-1, \lambda^*) \\
0 &= L_d(0, \lambda^*) \leq L_d(1, \lambda^*)
\end{align*}
$$

leads to $\lambda^* \geq 1$

Pick $\lambda^* = 1$

Saddle-point condition applies for $\lambda^{**} > \lambda^*$
Algorithm needs to look for $\gamma^{**} > \gamma^*$ and $\mu^{**} > \mu^*$

Outer Loop

$L_m(z, \gamma, \mu) \uparrow_{\gamma, \mu}$ to find $\gamma^{**}, \mu^{**}$

Inner Loop

$L_m(z, \gamma, \mu) \downarrow_z$ to find $z^*$

PARTITIONING OF ESPC FOR SEPARABLE NEIGHBORHOODS
Separable Neighborhoods

\( \mathcal{N}_p(z) \) (mixed neighborhood of path \( z = (z(0), \ldots, z(N + 1))^T \)) is the union of mixed neighborhoods in each stage, while keeping the path fixed in other stages.

Path \( z \) is a constrained local minimum in mixed space \( (CLM_m) \) iff

- \( z \) is feasible
- No feasible path in \( \mathcal{N}_p(z) \) has better objective value than \( J(z) \)

Decomposition of Lagrangian Function into Stages

Decompose Lagrangian function

\[
L_m(z, \gamma, \mu) = J(z) + \sum_{t=0}^{N+1} \gamma^T(t) \cdot |E(z(t))| + \mu^T \cdot |I(z)|
\]

into distributed Lagrangian function for stage \( t, t = 0, \ldots, N + 1, \)

\[
\Gamma_m(z, \gamma(t), \mu) = J(z) + \gamma(t) \cdot |E(z(t))| + \mu \cdot |I(z)|
\]
Distributed Necessary & Sufficient ESPC for $CLM_m$

- Path $z$ is a $CLM_m$ if and only if it satisfies

  - Distributed Necessary & Sufficient ESPC for all $t = 0, 1, \cdots, N + 1$

    $$\Gamma_m(z^*, \gamma(t)', \mu^*) \leq \Gamma_m(z^*, \gamma(t)^*, \mu^*) \leq \Gamma_m(z', \gamma(t)^*, \mu^*)$$

    $$L_m(z^*, \gamma^*, \mu) \leq L_m(z^*, \gamma^*, \mu^*)$$

    for all $z' = (z(0), \ldots, z(t - 1), z(t)', z(t + 1), \ldots, z(N + 1)) \in \mathcal{N}_p^{(t)}(z^*)$

  - Condition is also true for all $\gamma(t)^{**} > \gamma(t)^*$ and $\mu^{**} > \mu^*$

Reduced Search Space for Finding Feasible/Optimal Paths

Significant reduction in complexity
Iterative Implementation

\[ L_m(z, \gamma, \mu) \uparrow_{\mu} \text{ to find } \mu^{**} \]

\[ \Gamma_m(z, \gamma, \mu) \uparrow_{\gamma_N(0)} \text{ to find } \gamma^{**}(0) \]

\[ \Gamma_m(z, \gamma, \mu) \downarrow_{z(0)} \]

\[ \Gamma_m(z, \gamma, \mu) \uparrow_{\gamma_{N+1}} \text{ to find } \gamma^{**}(N+1) \]

\[ \Gamma_m(z, \gamma, \mu) \downarrow_{z(N+1)} \]

Observation:

- Based on a separable neighborhood, the combined local minimum of \( \Gamma_m \) in all subspaces is the local minimum of \( L_m \).

Equivalent Search:

\[ L_m(z, \gamma, \mu) \uparrow_{\mu} \text{ to find } \mu^{**} \]

\[ L_m(z, \gamma, \mu) \uparrow_{\gamma} \text{ to find } \gamma^{**} \]

\[ L_m(z, \gamma, \mu) \downarrow_{z} \text{ to find } z^{*} \]

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EXPERIMENTAL RESULTS ON MIPS
MIPS+DIS Algorithm

1. procedure MIPS+DIS
2. compute the relevant actions for each goal fact;
3. compute the partial orders among goal facts;
4. generate an initial ordered goal list of goal facts;
5. set \( \text{iter} \leftarrow 0 \);
6. repeat
7. for each goal fact in the goal list
8. call modified MIPS to solve the subproblem;
9. end_for
10. if (feasible plan found)
11. call PERT to generate & evaluate a parallel plan;
12. decrease some Lagrange multipliers;
13. else increase Lagrange multipliers \( \gamma \) on unsatisfied global constraints;
14. \( \text{iter} \leftarrow \text{iter} + 1 \);
15. if (\( \text{iter} \% \tau == 0 \)) dynamically re-order the goals;
16. until no change on \( z \) and \( \gamma \) in an iteration;
17. end_procedure

Search-space reduction for a subproblem

- For each goal fact
  - Backward relevance analysis to get a relevance list of relevant actions

- Possible improvement: tighter reduction
Ordering of goals

- Difficult goals be resolved before easier ones
- Two levels of partial ordering:
  1. Ascending number of irrelevant actions
  2. Descending minimum number of preconditions
- Example: \((\text{at person3 city1}) \rightarrow (\text{at truck1 city2}) \rightarrow (\text{at driver2 city1})\)
- Possible improvement
  - Heuristic distance from current state to each goal
  - Dynamic ordering during search

Modified MIPS

- Modify heuristic function for \(A^*\) search

\[
H'(s) = H(s) + D(s) + \sum_{i=1}^{NG} \left( \gamma_i a_i + \zeta_i h_i \right),
\]

- Prune nodes not in relevance list at node expansion
**Heuristic objective**

- A heuristic objective $D(s)$ to measure solution quality:

$$D(s) = \alpha_D \ast n_d,$$

(2)

- $\alpha_D$ is a weighting factor (0.01 in our experiments),
- $n_d$ is the number of action dependencies in the relaxed plan from $s$ to goal
- Possible improvement:
  - Apply PERT and compute the objective function at each $s$

**Lagrange Multipliers**

- If a feasible plan is not found:
  - Increase Lagrange multipliers of those unsatisfied goal facts
- If a feasible plan is found:
  - Call PERT to generate parallel plan and evaluate quality
  - Decrease some Lagrange multipliers
- Possible improvement:
  - May periodically decrease Lagrange multipliers even when a feasible plan is not found
Experimental Results

- 140 problems in 7 domains

- 75 problems solvable by MIPS in 1 second:
  - MIPS+DIS improves 67 in solution quality

- 43 problems solvable by MIPS in $10^3$ seconds:
  - 80% better trade-off in time and quality by MIPS+DIS

- 22 problems unsolvable by MIPS in $10^3$ seconds:
  - 15 solvable by MIPS+DIS in $10^3$ seconds:
Conclusions

- Partitioning of mixed constrained optimization in temporal planning
  - Distributed method to resolve global constraints across partitions
  - Significant reduction in search space by reducing the base of the exponential complexity
- Few parameters to tune in algorithm
- Significant improvement on PDDL2.1 planning problems